BEHAVIOUR AND MODELLING OF REINFORCED CONCRETE SLABS AND SHELLS UNDER STATIC AND DYNAMIC LOADS

by

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A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy Graduate Department of Civil Engineering University of Toronto

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ABSTRACT

A procedure for improved nonlinear analysis of reinforced concrete (RC) slab and shell structures is presented. The finite element program developed employs a layered thick-shell formulation which considers out-of-plane (through-thickness) shear forces, a feature which makes it notably different from most shell analysis programs. Previous versions were of limited use due to their inabilities to accurately capture out-of-plane shear failures, and because analyses were restricted to force-controlled monotonic loading conditions. The research comprising this thesis focuses on addressing these limitations, and implementing new analysis features extending the range of structures and loading conditions that can be considered.

Contributions toward the redevelopment of the program include: i) a new solution algorithm for out-of-plane shear, ii) modelling of cracked RC in accordance with the Disturbed Stress Field Model, iii) the addition of fibre-reinforced concrete (FRC) modelling capabilities, and iv) the addition of cyclic and dynamic analysis capabilities. The accuracy of the program was verified using test specimens presented in the literature spanning various member types and loading conditions. The new program features are shown to enhance modelling capabilities and provide accurate assessments of shear-critical structures.

An experimental program consisting of RC and FRC slab specimens under dynamic loading conditions was performed. Eight intermediate-scale slabs were constructed and tested to failure under sequential high-mass low-velocity impact. The data from the testing program were used to verify the dynamic and FRC modelling procedures developed, and to contribute to a research area which is currently limited in the database of literature: the global response of RC and FRC elements under impact. Test results showed that the FRC was effective in increasing capacity, reducing crack widths and spacings, and mitigating local damage under impact.

Analyses of the slabs showed that high accuracy estimates can be obtained for RC and FRC elements under impact using basic modelling techniques and simple finite element meshes.

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CHAPTER 1: INTRODUCTION

1.1 Background

Reinforced concrete shell structures comprise much of the world's infrastructure. Shell-type construction methods have been employed for over 2000 years and can be found in some of the most iconic and state-of-the-art structures built throughout history. Today, computer-based analytical procedures are commonly used in the design of reinforced concrete structures. In the case of reinforced concrete shells which are often characterized by curvilinear geometries and complex loading schemes, the use of computational modelling procedures is particularly appealing and may often provide a practical approach toward the design of these types of structures. Examples of complex reinforced concrete shell structures include:

- Offshore construction applications which, in addition to the gravity loads of the superstructure, must resist extreme water pressures, forces exerted from waves, and extreme dynamic loads arising from potential vehicle or iceberg impact.
- Storage silos and container structures subjected to combined in-plane and out-of-plane stress states arising from non-uniform geometric conditions and/or non-uniform loading conditions.
- Nuclear containment structures which, in the event of an emergency, must resist combined thermal loading and internal pressures, and are required to do so while satisfying strict serviceability criteria to prevent the escape of hazardous materials. Containment structures are also required to function as protective barriers, shielding internal reactors from external threats such as impact or blast loads.
- Thin-shell structures which often possess curvilinear, and often irregular, geometries are designed to distribute loads primarily through membrane action. However, as a result of their slender geometries, these types of structures can be sensitive to geometric nonlinearity effects and localized buckling phenomena.

Further complexities arise when these structures are constructed in regions where severe environmental conditions or seismicity are relevant. Evident from the loading requirements for the structures noted above, the availability of advanced analytical tools would be highly advantageous for the design of such structures.

To facilitate the design of complex reinforced concrete structures, linear elastic finite element analyses are often performed to evaluate the loading conditions experienced by individual structural elements, and the elements are then designed in accordance with codified provisions or with the aid of supplemental analytical tools. Although this procedure certainly forms a rational design approach, linear elastic finite element programs do not consider the redistribution of internal forces that can occur due to local changes in stiffness arising from cracking or crushing of concrete, yielding of reinforcement, or second-order mechanisms which may significantly influence the behaviours of reinforced concrete structures (e.g., post-cracking dilation, local unloading behaviours, etc.) and, as a result, the computed member force demands. Also relevant to the problem is the manner in which out-of-plane (i.e., through-thickness) shear forces are accommodated. Most commercial shell analysis programs, both linear elastic as well as those that consider material nonlinearities, tend to neglect out-of-plane loading scenarios cannot be estimated using such programs.

Designing reinforced concrete structures to withstand blast and impact loads has traditionally been approached in a highly idealized manner, with procedures typically consisting of empirical formulas used to estimate member damage levels or capacity (Sliter, 1980; Kishi et al., 2002) and simple macro-models which reduce structural members to single-degrees-of-freedom (UFC 3-340-02, 2008). Although the simplicities of such methods make them appealing, they have been shown to be unreliable (El-Dakhakni et al., 2009; Chen and May, 2009) and they provide limited information regarding the actual dynamic response and post-event state of the structure.

As modern code provisions continue to evolve toward performance-based design methodologies, and as extreme loading scenarios such as impact and blast are considered in the design process more regularly, the need for analytical tools which are capable of accurately modelling the behaviours of complex reinforced concrete structures under a wide range of loading conditions continues to grow.

1.2 Research Motivation

Great strides have been made in the past several decades with respect to the development of reinforced concrete analysis and modelling procedures. These advancements are in large part due to the significant amount of research dedicated to understanding the behaviour of reinforced concrete subjected to shear (Collins et al., 2008), a complex design problem which has been responsible for numerous catastrophic failures. One such research effort undertaken at the University of Toronto led to the formulation of the Modified Compression Field Theory (MCFT) (Vecchio and Collins, 1986), a rational model which has been shown to be capable of estimating the behaviour of reinforced concrete under shear. Today the implementation of advanced behavioural models such as the MCFT in the development of nonlinear analysis programs continues to be a major focus of research, as it has been demonstrated that such models can be used to approach the design and assessment of complicated reinforced concrete structures in a rational manner.

Several commercial programs have been developed to analyze reinforced concrete under general loading conditions, including blast and impact. However, these tools are almost entirely confined to hydrocodes, and often such approaches have resulted in limited successes as they typically require complex micro-modelling representations of the structure or element under consideration which is expensive in preparation and computation, and many of the available commercial programs have shown deficiencies in their abilities to capture the responses of shear-critical elements. Some researchers have suggested that further analytical advancement in the areas of blast and impact has been hindered by a lack of high-quality experimental data (Chen and May, 2009). As such, the development of alternative analytical tools which are practical, employ rational modelling approaches capable of capturing the behaviour of shear-critical structures, and are capable of analyzing reinforced concrete structures under general loading conditions represents a research area which is both significant and relevant in the design of modern structures.

1.3 Study Scope and Objectives

The primary focus of the research program presented in this thesis is aimed toward further development of the software program VecTor4: a nonlinear finite element analysis program dedicated to the analysis of reinforced concrete slab and shell structures. The software program

represents the redevelopment of program 'APECS' (Polak and Vecchio, 1993b), a shell analysis program based on the formulations of the MCFT. The goal of the analytical research program is to develop a tool which is generally applicable for the analysis of reinforced concrete shell structures under different types of loading conditions. As most commercial programs tend to neglect shear, or have demonstrated an inability to capture brittle concrete behaviour, the tool should be developed on the basis of an advanced behavioural model which is capable of capturing shear-critical behaviour. As a secondary analytical objective, the ability to analyze structures comprised of steel fibre reinforced concretes (SFRC) coupled with conventional reinforcing steel (often identified as R/FRC) should also be incorporated, as R/FRC is emerging as a new construction technology and its use is particularly appealing in the design of reinforced concrete shells, and in structures subjected to extreme loads.

An experimental testing program focused on investigating the behaviour of RC and R/FRC slabs subjected to impact loading conditions forms the second aspect of the research program. The test program was performed to address the lack of 'high-quality' data pertaining to reinforced concrete under extreme loads, and to provide data pertaining to an emerging research area where currently only limited data exist: the global response of R/FRC structures under impact.

The research program presented in this thesis can be subdivided into the following four objectives:

- 1) To identify and correct deficiencies pertaining to the existing version of the software program VecTor4. Specific tasks include:
- Modification of the 'thick-shell' formulation.
- Development of subroutines to evaluate local stress conditions at the crack.
- Implementation of a new solution algorithm to enhance stability.
- 2) To implement and verify the performance of new general features and analysis options which extend the range of structure-types, loading conditions, and behavioural mechanisms that can be considered within the VecTor4 analyses. New features include:
- The Disturbed Stress Field Model (DSFM) (Vecchio, 2000); an extension of the MCFT.

- The Simplified Diverse Embedment Model (SDEM) (Lee et al., 2013); a model used to compute the tensile response of cracked steel fibre reinforced concrete.
- Second-order effects such as dowel action, post-cracking Poisson's effect/dilation, concrete prestrains (i.e., shrinkage), etc.
- · Shell element centerline offset capabilities.
- The addition of truss-bar elements.
- Out-of-plane shear strength enhancements to approximately account for D-regions.
- Displacement-controlled analyses.
- 3) To develop and verify the performance of cyclic and dynamic analysis capabilities within VecTor4. Required implementations include:
- Concrete hysteresis models.
- An element mass lumping scheme suitable for complex high-order finite elements.
- Subroutines to evaluate the supplemental viscous damping matrix.
- Material strain rate effects for concrete and steel reinforcement.
- Direct time integration methods which are compatible with the general solution method employed by VecTor4.
- Dynamic load vectors to accommodate ground accelerations, predefined impulses, and mass impact loading conditions.
- 4) To design and carry-out an experimental testing program focused on the behavior of RC and R/FRC slabs subjected to drop-weight impact loading conditions, with a specific focus on:
- Assessing the applicability of using R/FRC elements in impact-resistant design.
- Adding to a limited database pertaining to the global response of reinforced concrete structures under impact.
- Providing data in a research area where little or no testing has been performed: the global response of R/FRC elements under impact.

1.4 Thesis Contents

The second chapter of this thesis presents background information pertaining to the two principal topic areas forming this research program: i) the nonlinear analysis of reinforced concrete shells and ii) the experimental testing of reinforced concrete elements under impact loading conditions.

The information regarding (i) is provided primarily for context, and that regarding (ii) was considered in the development of the experimental portion of the research program.

Chapter 3 presents the specimen details and testing methodologies used in the experimental program. The instrumentation and data measurement techniques employed are summarized. The test results from the experimental program are presented and discussed in Chapter 4. Additionally, an assessment of the acquired digital data set is provided.

Chapter 5 provides information regarding the formulation of the layered shell finite elements employed in the nonlinear analysis program VecTor4. Brief summaries of the MCFT and DSFM behavioural models are provided.

Chapter 6 explains the implementation of the new general loading features and improved solution methods in VecTor4. The analytical results from a relatively extensive monotonic loading verification study are presented, and the adequacy of the results is discussed.

Chapter 7 provides an overview of the new subroutines which were added to incorporate dynamic analysis capabilities within VecTor4. The nonlinear dynamic procedures were verified using data from the experimental program undertaken as well as data from other dynamic tests presented in the literature.

Lastly, Chapter 8 presents the conclusions from the experimental and analytical studies, and provides recommendations for future investigations.

CHAPTER 2: LITERATURE REVIEW

This chapter provides an overview of previously developed analytical procedures and experimental investigations which are relevant to the work undertaken in this thesis. The information in this chapter is not intended to be exhaustive, but rather is included to provide context regarding approaches which have been used to analyze reinforced concrete slab and shell structures, and to provide a brief summary of previous analytical and experimental investigations focused on the assessment of reinforced concrete elements subjected to extreme loading conditions.

2.1 Analysis of RC Shells

Early analytical investigations pertaining to reinforced concrete shells were primarily focused on developing design procedures for elements subjected to in-plane membrane forces. Although it was well established that bending stresses occur in nearly all forms of shells, including those designed to carry loads predominantly as membranes, uncertainties regarding the design procedures for reinforced concrete membranes under in-plane shear, at least to some degree, suppressed the development of generalized shell modelling techniques (Gupta, 1984).

2.1.1 Layered Models

The introduction of multilayer, or stacked membrane, models represents a significant advancement toward what is currently the state-of-the-art in generalized modelling techniques for reinforced concrete shells. By subdividing the shell into a series of layers, and treating each layer as if it behaves as an individual membrane with uniform in-plane stress and strain conditions, stiffness variations through the thickness of the element resulting from different types of materials or material nonlinearities can be represented discretely. With the use of appropriate compatibility assumptions to describe the in-plane strain variation through the thickness of the shell (plane sections remain plane, for example), the analytical approach to the problem remains essentially the same as that employed in the case of a two-dimensional membrane. The in-plane stresses, and bending and twisting contributions (M_x , M_y , M_{xy}) are computed from the moments of the in-plane normal stresses and the in-plane shear stresses, respectively. The layered shell concept and the orientation of the sectional forces are illustrated in Figure 2.1.



(c) bending and membrane sectional forces Figure 2.1 – Multilayer Shell Element (adapted from Collins and Mitchell, 1997)

Hand et al. (1973) reported early applications of layered shell finite elements in the nonlinear analysis of reinforced concrete structures. Four-node rectangular shell elements with twenty degrees-of-freedom per element (five dofs per node: u, v, w, $w_{,x}$, $w_{,y}$) were considered. Kirchhoff plate assumptions (i.e., plane sections remain plane) were used to develop membrane and bending constitutive relations. Concrete was modelled as a tri-linear elastic-perfectly plastic material with consideration of a biaxial compression yield criterion (Kupfer et al., 1969). Post-crack concrete tensile stresses were neglected and the in-plane shear stiffness of the concrete, which the authors attributed to aggregate interlock and dowel action, was estimated using a shear retention approach. The developed finite element program was used to compute the load-deflection behaviours from experimental tests reported in literature: rectangular specimens under bending and/or torsion, a two-way slab specimen, and a funicular shell. In general, varying levels of agreement were obtained between the computed responses and the experimental results.

However, the authors' analytical responses did show improvement over most results reported by others at that time. Lastly, the authors reported uncertainties regarding appropriate values for the shear stiffness retention factor, but noted that its inclusion was required to maintain stability in the analyses.

Following the work of Hand et al. (1973), Lin and Scordelis (1975) developed a nonlinear RC shell finite element analysis program using three-node triangular elements. Similar elastic-plastic relations were used to model the compressive response of concrete, and the use of a shear retention factor to incorporate the concrete shear stiffness was also considered. However, notably different from the approach employed by Hand et al., the authors included a post-cracking tensile response to account for concrete tension stiffening based on the average stress concept reported earlier by Scanlon (1971). The authors suggested that in the case of typical RC shell structures, assumptions pertaining to the tension behaviour of the concrete were generally more relevant than those pertaining to compression. The finite element program was verified using experimental data reported in the literature. The authors found that the inclusion of tension stiffening significantly influenced the post-cracking response of under-reinforced concrete structures, but had little effect on the ultimate capacities. Lastly, in agreement with that reported by Hand et al., the authors found that further investigation regarding appropriate selection of the shear retention factor was required.

Schnobrich (1977) reported general findings based on a review of finite element modelling techniques developed for reinforced concrete. Selection of material models, the treatment of cracked concrete in tension, and uncertainties regarding the computation of the concrete shear modulus were topics included in the discussion. Additional discussion regarding the application and limitations of layered elements in the analyses of plate and shell structures was also provided. Schnobrich noted that for structures dominated by flexure and/or membrane loads, the types of layered shell elements employed by Hand et al. (1973) and Lin and Scordelis (1975) were perhaps the most suitable method for analyzing complicated RC structures. However, if out-of-plane (through-thickness) shear was relevant in the problem, alternative three-dimensional modelling techniques should be employed. Schnobrich used the example of an analysis to investigate the punching behaviour of a floor plate surrounding a column (see Figure 2.2).



Figure 2.2 – Model to Account for Shear (adapted from Schnobrich, 1977)

The use of high-order three-dimensional solid finite elements immediately surrounding the columns, with transition elements linking the shells to the solids, was suggested as one method which might be used to approach the problem. However, it was noted that mechanisms which were still not well understood at the time, such as aggregate interlock and dowel action, would likely have to be incorporated in a highly artificial manner since their extension to three-dimensional applications hadn't yet been investigated. Additionally, the development of reasonable triaxial stress-strain relations would also be required.

2.1.2 Out-of-Plane Shear

Layered RC shell analysis procedures have traditionally been developed based on the assumption that out-of-plane shear forces are negligible. This methodology was used for the layered models presented in the previous section, and currently forms the basis of most commercial RC shell analysis programs. In the design of complex RC structures which experience combined bending, membrane, and out-of-plane shear forces, it is common to use shell analysis procedures to evaluate the membrane and bending contributions of the response, and rely on supplemental tools or provisions to address the out-of-plane shear contributions. However, the validities of these types of approaches are somewhat unfounded as most of the supplemental procedures used to compute the out-of-plane shear strength are empirical and were developed for much simpler beam-type elements (Adebar, 1989; Collins and Mitchell, 1997). Additionally, the uncoupling of the in-plane and out-of-plane response contributions, particularly in the case of thick-shell structures, is likely to deviate from the actual behaviour of RC shells under combined in-plane and out-of-plane loading conditions.

Consider the example of the inverted cone RC storage silo presented in Figure 2.3. For the purpose of enhancing flow conditions and reducing the occurrence of material blockage during operation, these types of silos are constructed such that material is discharged eccentrically at the base of the inverted cone hopper along the perimeter of the silo wall. As a result of this eccentric discharge condition, large out-of-plane shear forces and bending moments surrounding the discharging material develop locally in the wall of the silo (see Figure 2.3b).



Modern Eurocode provisions (EN 1991-4: 2006) provide guidance for estimating the nonuniform stress distributions acting on the walls of these silos; however, designers are still faced with the challenge of designing for three-dimensional loading conditions consisting of combined in-plane and out-of-plane force contributions. Furthermore, because cylindrical silo structures rely heavily on the development of hoop tension to resist lateral storage loads, accurate estimates of the silo's ability to resist combined in-plane tension and out-of-plane shear are required.

Prior to the development of the current Eurocode provisions (EN 1991-4: 2006), many of these inverted cone silos were designed as non-prestressed structures, contained little or no out-of-plane shear reinforcement, and were designed without consideration of developed shear forces and bending moments surrounding the material discharge regions (McKay, 2006). As such, RC

shell analysis tools which are capable of providing accurate assessments of structures subjected to combined in-plane and out-of-plane loading conditions would not only serve as a valuable design aid, but are also required to assess the adequacy of existing inverted cone storage silos.

Alternative modelling methods, such as the finite element approach illustrated by Schnobrich (1977) which used three-dimensional solid elements in place of shells in regions where out-ofplane shear was expected to be significant, are somewhat impractical as they require complicated modelling procedures with the use of highly-specialized finite elements. As such, a significant amount of research has been undertaken in an effort to develop analysis procedures which are applicable for the analysis of RC shells under combined membrane (N_x, N_y, N_{xy}), bending (M_x, M_y, M_{xy}), and out-of-plane shear forces (V_{xz}, V_{yz}). A reinforced concrete shell element under the eight possible sectional forces which can be considered in these types of analysis procedures is presented in Figure 2.4.



Figure 2.4 – Shell with Out-of-Plane Shear (adapted from Collins and Mitchell, 1997)

Owen and Figueiras (1984) were among early investigators to consider out-of-plane shear in the analysis of RC shells. The authors developed a nonlinear finite element program which employed a 'thick-shell' layered heterosis element and considered geometric nonlinearity. The finite element was developed from general three-dimensional elasticity and strain compatibility assumptions based on Reissner-Mindlin theory (Reissner, 1945; Mindlin, 1951) (i.e., plane sections remain plane, but not necessarily normal to the midsurface) which resulted in constant out-of-plane shear strain distributions through the thickness of the element (see Figure 2.5a). Nonlinear material behaviour was done in accordance with a smeared rotating crack model which considered a series of plastic flow rules to account for yielding and crushing criteria. The

program considered concrete cracking, the influence of tension stiffening, and cracked shear moduli which were updated throughout the analysis based on the current state of the principal tensile strain. The nonlinear finite element analysis program was shown to provide good agreement with experimental tests of RC slabs and cylindrical shell structures.

A sectional analysis tool developed by Kirschner and Collins (1986) for RC shells under general loading conditions used the relationships of the MCFT (Vecchio and Collins, 1986) to model the behaviour of cracked reinforced concrete. As such, influential mechanisms such as tension stiffening, compression softening, local behaviour at the crack locations, and the influence of variable crack widths were considered in the analyses. However, it should be noted that for simplicity, the local 'crack-check' was neglected in elements subjected to three-dimensional stress conditions. The model was developed on the assumptions that plane sections remain plane, and that the out-of-plane shear stresses are uniform throughout the core layers of the shell but are equal to zero at the top and bottom surfaces (see Figure 2.5b). As is typically the case in shell analyses, it was assumed that the normal forces in the out-of-plane direction were negligible; however, by treating out-of-plane shear reinforcement as an inherent property of the concrete layers, contributions from the shear reinforcement were considered in the analysis. The analytical tool was verified against experimental data of reinforced shell elements subjected to combined membrane loads and bending. The influence of out-of-plane shear forces was investigated analytically.



Figure 2.5 – Sectional Analysis Assumptions

Adebar and Collins (1989) used the Shell Element Tester developed at the University of Toronto to perform a series of tests on RC shells subjected to combined in-plane and out-of-plane shear, a research area where essentially no experimental data had existed prior to this program (see

Figure 2.6). The experimental data were used to verify the performance of a sectional analysis tool developed by the same authors. The analytical tool was similar to the program developed by Kirschner and Collins in that it was based on the formulations of the MCFT and considered all eight sectional force contributions; however, the computational approach used to incorporate out-of-plane shear forces was significantly different. Rather than assuming a distribution of the out-of-plane shear strains or the out-of-plane shear stresses, the analysis method employed a simplified technique which computed the response of the shell subject to in-plane loads only, and applied an in-plane correction to account for out-of-plane shear stresses. The benefit of this approach was that it only required a three-dimensional analysis to be performed at the mid-height of the shell and, as such, reduced computation time significantly. Additionally, the local 'crack-check' feature of the MCFT which was originally omitted in the work of Kirschner and Collins was extended to three-dimensional applications and was included in the work of Adebar and Collins.



Figure 2.6 – Shell Element Tester (University of Toronto)

Polak and Vecchio (1993a) modified the nonlinear layered shell finite element program developed by Owen and Figueiras (1984) such that the modelling of cracked reinforced concrete was done according to the formulations of the MCFT. Additionally, the ability to consider out-of-plane shear reinforcement within the analysis procedure was added to the program. The

underlying theory of the shell element was unchanged from that reported by Owen and Figueiras and, as such, the strain compatibility assumptions were based on the governing Reissner-Mindlin plate theory: plane sections remain plane but not necessarily normal to the mid-surface of the shell element, resulting in an out-of-plane shear strain distribution which is constant through the thickness of the shell. The analytical work was complimented with an experimental program consisting of RC shells subject to combined in-plane shear and bending (including biaxial bending scenarios), tested using the University of Toronto's Shell Element Tester.

Similar shell finite elements developed on the bases of Reissner-Mindlin theory have been used by Maekawa et al. to analyze a wide range of RC structures such as plate and slab elements (Song et. al, 2002), as well as three-dimensional cylindrical tank and silo structures (Maekawa et al., 2003) under monotonic and reversed cyclic loading conditions. The analyses were performed using a series of multidirectional smeared fixed crack approaches, employing constitutive relations developed by Maekawa et al. (2003), to model the behaviour of cracked reinforced concrete. Using this approach, the authors achieved high levels of agreement with experimental results for both monotonic and cyclic loading scenarios; however, the majority of the test specimens used to verify the developed analysis procedures were governed by in-plane failure mechanisms in which out-of-plane shear force contributions were generally minimal.

Simplifying assumptions regarding the out-of-plane shear stress or shear strain behaviour of RC shells are commonly used to develop shell finite elements or shell analysis procedures which consider out-of-plane forces. In the literature summarized above, all of the models were based on the assumption that plane sections remain plane, and that out-of-plane normal stresses (i.e., clamping stresses) were negligible. Models which accounted for out-of-plane shear behaviour also considered additional assumptions: finite elements developed on the bases of Reissner-Mindlin theory assumed that the out-of-plane shear strains of the element were constant through the thickness, the sectional analysis tool developed by Kirschner and Collins (1986) assumed that the out-of-plane shear stress was constant through the thickness of the shell but equal to zero at the free surfaces, and the sectional analysis tool developed by Adebar and Collins (1989) assumed that the error computed at the shell's midheight (error attributed to neglecting out-of-plane shear in the computation of the in-plane stress conditions) was representative of the error over the thickness of the shell.

Some researchers have developed alternative analytical methods which do not require broad generalized assumptions regarding the out-of-plane shear response. Bentz (2000) developed a series of analysis programs based on the formulations of the MCFT which included a sectional analysis tool for three-dimensional RC shell elements. The program uses a layered approach and is based upon the assumptions that plane sections remain plane, and that out-of-plane clamping stresses are negligible. However, in contrast to the approaches noted above, the solution method does not require any assumptions regarding the out-of-plane shear stress or the out-of-plane shear strain distributions. Rather, the program uses a solution technique referred to as the longitudinal stiffness method to explicitly compute the out-of-plane shear response. Derivatives of the in-plane layer stresses taken with respect to in-plane strains form the basis of computing an estimate of the out-of-plane shear strains within each layer of the shell. The method is iterative, but has been shown to be stable and avoids the use of generalized assumptions regarding the out-of-plane shear response. Bentz employed the same methodology in the development of a two-dimensional sectional analysis program and showed that the program was capable of providing high accuracy estimates of the behaviours of shear-critical reinforced concrete elements. A similar approach employing a different numerical technique (Schulz and d'Avila, 2010) has also been reported, and has been shown to provide reasonable out-of-plane strength estimates of the RC shell specimens tested by Adebar and Collins (1989).

As summarized in this section, a relatively wide range of solution procedures have been employed in the analysis of RC shells. Although out-of-plane shear contributions are quite often neglected in shell analyses, in many scenarios the out-of-plane behaviour can significantly influence, or even govern, the responses of RC shell structures. Several analytical methods which consider out-of-plane shear forces have been developed. The solution methods range from finite elements which, to ease implementation and element development, typically provide coarse approximations of the actual out-of-plane shear strain distributions through the thickness of RC shell elements, to more sophisticated sectional analysis tools which provide explicit methods for computing the out-of-plane shear strain and shear stress distributions. However, to date, the use of the more sophisticated procedures have essentially been limited to single element sectional analysis techniques as they are significantly more costly in terms of computation, and their implementation within generalized shell finite element procedures presents several challenges. The primary obstacle being that layered finite elements are typically developed on the basis of known, or assumed, strain conditions.

2.2 Concrete Under Impact

The response of reinforced concrete structures under impact can vary significantly depending on the specific nature of the event. The concrete structure (i.e., the target) and the impactor (commonly referred to as the missile) often both play roles in dissipating energy and, as a result, the velocity and the material properties of the impactor will influence the structure response.

Impact events can be generally subdivided into one of two categories: 1) 'hard' impacts and 2) 'soft' impacts. Soft impacts refer to events in which the impactor itself experiences significant deformations over the course of the event, and hard impacts are classified as events in which the impactor experiences very little deformation relative to the deformations of the target structure. In either impact scenario, a reinforced concrete target may respond in several ways: i) the structure response is limited to the local impact region and the majority of the impact energy is dissipated through local damage mechanisms, ii) the impact energy is translated into global structural deformations (e.g., bending, shear, axial deformations) of the target, or in many cases iii) the impact energy is dissipated through a combination of local damage mechanisms and structure deformations. Local damage typical of reinforced concrete structures under impact can be classified using three response categories of increasing severity: 1) localized damage is in the form of mass penetration with the occurrence of limited concrete spalling on the impact-surface, 2) penetration depths exceeding the spalling crater and the occurrence of concrete scabbing from the back surface of the target, and 3) impactor perforates the target structure and exits the back surface with residual velocity. It is typically assumed that the severity of the local damage is primarily dependent upon the velocity of the impactor (Kennedy, 1976). Local impact damage mechanisms are illustrated in Figure 2.7.

The majority of impact-related investigations performed to date have primarily focused on developing simple design criteria to limit the severity of local damage mechanisms attributed to high-velocity hard impact conditions. This is in large part due to the fact that the military and the nuclear industry were the main driving forces behind most of the early research work, and their focus was aimed at developing structures to resist loads from ballistic missiles and aircraft
impacts. As a result, much of the literature consists of empirical design formulae which are applicable to a very limited range of impact conditions. The use of these formulae to provide structure damage estimates for impact loading conditions outside of this range, such as high-mass low-velocity impacts, has been met with limited success (Chen and May, 2009).



The behaviour of RC structures under impulse and impact loading conditions is an area of research which is still not well understood; however, it continues to be motivated by a growing number of structures requiring procedures for impact assessment. Examples include structures designed to resist accidental loading scenarios such as falling rock impact, vehicle or ship collisions with buildings, bridges, or offshore structures, and structures which are commonly used in high-threat applications such as military fortification structures or high-hazard nuclear containment structures. For all of these applications, the global structure response must be considered in the design process as it may often govern failure.

2.2.1 Global Response of RC Slabs and Shells

This section presents some of the research undertaken to investigate the global dynamic response of reinforced concrete slabs and shells under impact. It should be noted however, that because local damage mechanisms are much more prevalent in RC slab-type elements, the majority of the work carried out to assess the global response have focused primarily on the behaviour of RC beams under impact. As such, several of the studies presented in this section could also be considered relevant in the context of local impact response behaviour. Analytical as well as experimental research programs have been included.

Rebora et al. (1976)

Rebora et al. were among the first to develop nonlinear finite element procedures which were capable of analyzing reinforced concrete systems at the structure level under extreme impact conditions. The authors developed a half-structure model of a nuclear containment shield building and applied a localized impulse load to represent the horizontal impact of an aircraft (Boeing 707-320) on the outer surface. High-order isoparametric solid elements were used to represent the concrete, and steel reinforcing bars were modelled using degenerated membrane elements on the interior and exterior surfaces of the structure. Triaxial constitutive modelling was used for the concrete elements with the premise that under tension, and low to moderate compressive stresses, linear constitutive relations were valid. However, under high compressive stresses nonlinear material laws which included strain rate effects (Saugy, 1969), and a triaxial yield criterion developed by Zimmerman (1975) were considered. Due to the limited computational capabilities available at the time, a relatively coarse mesh consisting of one element through the wall thickness was used to represent the structure (see Figure 2.8).



Figure 2.8 – Shell-Structure Geometry (Rebora et al., 1976)

The analysis was performed using a direct integration time-stepping procedure with no supplemental viscous damping. The authors reported crack pattern progressions on the interior and exterior surfaces of the shell and presented steel stress distributions. Numerical instabilities within the elements comprising the impact loaded region prevented the analysis from completing the full 350 millisecond impact load-time history. It was suggested that the noted instability was possibly due to the perforation of the structure wall within the impact region.

Miyamoto et al. (1991)

Miyamoto et al. (1991a) developed a nonlinear finite element analysis program using the layered plate element employed by Hand et al., previously summarized in Section 2.1.1. The authors used the triaxial failure criterion developed by Ottosen (1977) combined with a plasticity based theory to model the compressive response of concrete. The concrete layers were treated as plane stress elements; however, sectional out-of-plane shear stresses computed on the basis of the bending and twisting moments were used to modify the in-plane stress conditions. The distribution of the out-of-plane shear stress was assumed to be parabolic through the thickness of the plate. Similar to the work of Hand et al., a two-dimensional smeared crack approach with a constant shear stiffness value was considered in evaluating the equilibrium conditions of the concrete layers. Because the study was primarily focused on analyzing the response of RC slabs under soft impact loading conditions, the authors did not consider dynamic material properties or material strain rate modifiers. The finite element program was verified using test results from a series of RC slabs tested by the same authors. The slabs were 1,300 mm square, 130 mm thick, and were impacted at their centre-points using a pendulum-based falling-mass of 500 kg. Rubber pads on the surfaces of the slabs created the soft impact condition. Additional details regarding the experimental program are presented in a companion paper (Miyamoto et al., 1991b). The impact load-time response was measured using accelerometers on the drop mass, and noncontact displacement transducers were used to measure the slabs' deflections. The impact loaddisplacement behaviours of the slabs were estimated reasonably well from the finite element analyses, and the computed residual crack patterns on the bottom surface of the slabs were similar to those measured experimentally. The authors reported numerical instabilities during the unloading phases of the impacts, and no analytical results or experimental data pertaining to the free vibration responses or the residual deformations of the slabs were reported.

Saito et al. (1993)

An investigation carried out by Saito et al. was focused on the experimental behaviour and analytical modelling of reinforced concrete slabs and shells under concentrated loads applied at different loading rates: 3.0 m/s, 0.03 m/s, and $3.0 \times 10^{-5} \text{ m/s}$. The RC specimens included flat slabs, T-beams, composite beam-slab assemblies, and cylindrical shell specimens (see Figure 2.9).



Figure 2.9 – Specimens tested by Saito et al. (1993)

The authors reported that the load capacities increased with loading rate for all of the RC specimens. They also noted that the deformation behaviours and the governing failure modes changed with loading rate. In addition to the experimental program, the authors also performed nonlinear finite element analyses of selected flat slab specimens and the cylindrical shell specimens. In both cases, quarter-specimen models were created using layered flat quadrilateral shell elements. The layered elements were based on the plane sections assumption and neglected out-of-plane shear behaviour. The compressive behaviour of the concrete was modelled using a trilinear elastic-plastic relationship. Post-cracking tensile stresses were neglected, no details were provided regarding shear stiffness assignments for the concrete. The concrete material models were fit to material tests performed at the same loading rates as the slab and shell test specimens. The computed load-midpoint displacement behaviours reasonably estimated the experimental responses prior to the occurrence of shear dominated behaviour. The authors concluded that with appropriate material models, the behaviours of these RC elements under increased loading rates can be modelled successfully prior to the occurrence of shear failures using layered shells.

May et al.

Chen and May (2009) performed a series of high-mass low-velocity drop-weight impact tests on reinforced concrete slab specimens of varying geometries (see Figure 2.10). The primary purpose

of the experimental program was to provide a 'high-quality' digital data set which could be used to appraise current and future developed analytical models. The program was primarily focused on local impact conditions; however, instrumentation measuring the global target behaviour was also provided. Impact force, reinforcing bar strains, and specimen accelerations comprised the digital data set and were measured at an acquisition rate of 500 kHz. No explicit measurements of the support reactions or the specimen displacements were collected. Scattered levels of agreement were obtained between the local damage conditions of the slab (e.g., scabbing, perforation) and various empirical design formulae available in the literature (Sliter, 1980).







A complimentary study performed by Sangi and May (2009) modelled and analyzed the impact slabs tested by Chen and May using the finite element analysis program *LS-DYNA*. The typical finite element mesh developed for the smaller slab specimens required approximately 50,000 brick elements and 55,000 nodes. The mass impactor was modelled discretely using solid elements with rigid material assignments and a surface-to-surface contact was defined between

the impactor and the target slab. The constructed finite element mesh for one of the smaller slab specimens (Slab S3) is presented in Figure 2.11.

The authors noted that the software program offered a large number of material modelling options for the concrete and the reinforcement materials, the majority of which were difficult to define and required complex material parameters. As such, the authors selected material models which required the fewest parameters and were simple to define or reasonably estimate. Analytical results pertaining to the local damage and the impact force-time history of Slab S3 were compared with that measured in the experimental program. Good agreement was obtained for the local damage zone and penetration on the impact (i.e., top) surface of the slab; the scabbing on the bottom surface of the slab was estimated with less accuracy. Analyses considering different damage models and mesh variation resulted in similar estimates of the impact force-time history of the slab. In all cases, the peak impact force was overestimated by approximately 35 to 40 %; however, the response shape and impact durations matched reasonably well with the experimental data.



Figure 2.11 – Finite Element Mesh for Slab S3 (Sangi and May, 2009)

Kishi et al.

Kishi et al. (2009) performed an analytical investigation for a rock protection structure using the software program *LS-DYNA*. The purpose of the study was to evaluate the impact capacity and the general response characteristics of the protective structure which was designed in accordance with governing Japanese code provisions. A half-structure model was created which required approximately 400,000 nodal points and 450,000 three-dimensional solid elements (see Figure 2.12). Concrete was modelled as a perfectly elastic-plastic material with yielding defined by Drucker-Prager's criterion. The material response for the steel reinforcing bars was defined using

a bilinear-hardening rule, and the sand cushion layer on top of the roof slab was represented using a crushable foam model which was available within the software program. The analyses were performed with 5 % viscous damping assigned to the first mode of vibration. An impact mass of five metric tonnes with fall heights ranging from 20 m to 250 m were considered in the simulations.



(a) general view of structure
(b) half-structure finite element mesh
Figure 2.12 – Analysis of Rock Protection Structure (Kishi et al., 2009)

The results from the finite element analyses suggested that the rock protection structure satisfied all service and ultimate limit states with large margins of safety when analyzed under the design level impact. Furthermore, the investigators found that the addition of the sand cushion significantly reduced the stress levels in the reinforced concrete structure.

Kishi et al. (2011) performed a series of nonlinear finite element analyses to simulate the behaviour of rectangular RC slabs under drop-weight impact. The slabs were 2,000 mm square, 180 mm thick, and were impacted at their midpoints. The slabs were reinforced orthogonally on the tension-side faces of the specimens; no reinforcement was provided for negative bending. One of the main objectives of the testing program was to assess what effect, if any, varying the support conditions of the slabs had on the impact response. Three support conditions were considered: i) line supports on all four sides, ii) line supports on two opposite edges, and iii) two corner supports with a line support along the opposite edge. All supports were formed from

individual pins which were free to rotate about all directions, and clamps were provided at the corners of the slabs to prevent uplift. The slabs were impacted using a 300 kg mass with an impact velocity of 4.0 m/s. A schematic of the three test slabs is shown in Figure 2.13.



Figure 2.13 – Kishi et al. (2011) Slab Specimens

The finite element analyses of the slabs were performed using *LS-DYNA*. Depending on the symmetry conditions of the supports, either quarter-slab or half-slab models were constructed. The concrete slab, the impacting mass, and the support conditions (including the load measurement instrumentation) were modelled using eight-node solid elements and the steel reinforcing bars were modelled using two-node beam elements (see Figure 2.14).



Figure 2.14 – Finite Element Meshes (Kishi et al., 2011)

The quarter-slab models consisted of approximately 90,000 nodal points and 100,000 elements. Half-slab models required approximately 160,000 nodes and 175,000 elements. Contact surfaces were used to model the interactions between the impacting mass and the slab, and the slab and the support reactions. Damping was specified as 5 % for the first mode of vibration and was selected through the performance of preliminary trial slab analyses. Constitutive models for the concrete and steel were similar to those described above for the analyses performed in Kishi et al., 2009.

The impact force-time, total support reaction force-time, and displacement-time histories computed from the finite element analyses were in good agreement with the data from the tests. The peak amplitudes of the impact forces and midpoint displacements were somewhat underestimated; however, the response shapes and frequencies generally matched the experimental behaviours. The authors noted that the different boundary conditions used in the experimental program had little effect on the impact-force responses, but led to significant differences in the measured reaction-time histories and the residual crack patterns on the bottom surfaces of the slabs. The reported crack patterns from experiments are presented in Figure 2.15.



Figure 2.15 – Residual Experimental Crack Patterns; bottom surface (Kishi et al., 2011)

2.2.2 R/FRC Under Impact

Investigations regarding fibre reinforced concrete (FRC) slabs and shells under impact are limited. Most of the impact-related research to date has focused on the behaviour of FRC at the material level and, as such, experimental tests involving FRC elements coupled with conventional reinforcing steel (R/FRC) are scarce. The global response of R/FRC slabs and

shells under impact is an area of research in which very little data exists in the database of literature, particularly with respect to analytical investigations. The following subsection presents the findings from research programs undertaken to investigate the dynamic response of R/FRC slabs under impact. However, all of the impact specimens which are reported in this section were predominantly governed by local failure mechanisms due to the testing methods employed. Additionally, for clarity, some studies pertaining to FRC elements (i.e., elements not containing conventional reinforcement) are also summarized in this section.

Ong et al. (1999)

An experimental investigation performed by Ong et al. (1999a) focused on assessing the impact resistance of FRC slabs without conventional reinforcement, constructed using different types of fibres. The slabs were 1,000 mm square, 50 mm thick, and were subjected to repeated drop-weight impacts using a hemispherical impactor with a mass of 43 kg, dropped from a height of 4 m. Line supports were provided along the four sides, and the slabs were restrained vertically to prevent specimen uplift. Three fibre types were considered in the testing program: i) straight polyolefin fibres, ii) kuralon-cut polyvinyl alcohol (PVA) fibres, and iii) end-hooked steel fibres. The FRC slabs were constructed with fibre volume fractions, V_f , ranging from 0.5 % to 2.0 %. The properties of the fibres are summarized in Table 2.1.

Fibre Type	Shape	$l_f(mm)$	$d_f(mm)$	$f_{u,f}(MPa)$	$E_f(MPa)$
polyolefin	straight	50	0.63	275	2,650
polyvinyl alcohol	straight	12	0.20	900	29,000
Steel	end-hooked	30	0.50	1,275	200,000

Table 2.1 – Properties of Fibres used in Ong et al. (1999a)

The authors found that the steel fibre reinforced concrete slabs exhibited superior performance in terms of slab cracking characteristics, energy absorption, and resistance to shear plug formation when compared to the slabs reinforced with polyolefin and PVA fibres. The polyolefin fibres were reported to fail by a combination of fibre rupture and fibre pullout, whereas the PVA and steel fibres were reported to fail by fibre pullout only.

In a companion study performed by the same authors, the behaviour of R/FRC slabs under low-velocity impact was investigated (Ong et al., 1999b). The same testing frame and specimen

geometry was used in this study; however, only end-hooked steel fibres were considered. The slabs were orthogonally reinforced in the plane of the specimens with conventional 6.5 mm diameter reinforcing bars, and the FRC was constructed using the same end-hooked steel fibres reported in the previously discussed program (see Table 2.1 for fibre material properties). Two drop-weight conditions were considered: i) a 20 kg drop-weight with a hemispherical striking surface, dropped from a height of 1.5 m, and ii) a 20 kg drop-weight with a flat striking surface, dropped from a height of 4.5 m. One impact condition was used per slab, and impacts were repeated until failure occurred. From the experimental results, the authors found that the addition of steel fibres up to a volume fraction of 2.0 % increased the number of impacts to failure by at least seven times. The slab midpoint deflection was found to decrease with increasing fibre volume, and additional conventional in-plane steel placed as compression reinforcement was also found to increase the impact resistance, particularly in the case of the lower-velocity hemispherical impactor. Overall, it was reported that the addition of the steel fibres greatly reduced local damage attributed to penetration and concrete scabbing, reduced slab deflections, and increased the energy absorption, all of which resulted in significant increases in the impact capacities of the slabs.

Kurihashi et al. (2006)

Kurihashi et al. performed an experimental testing program consisting of R/FRC slabs subjected to drop-weight impact loading conditions. The R/FRC slabs were 2,000 mm square, 180 mm thick, and were orthogonally reinforced with D16 bars spaced at 150 mm on the tension faces of the specimens. The R/FRC slabs were constructed using PVA (polyvinyl alcohol) short-fibres with volume fractions, V_{f_5} ranging from zero (i.e., conventional RC slab) to 3 %. In addition to the impact testing program, static load tests were also performed to evaluate the behaviours of the slabs under conventional testing conditions. Under static loading conditions, all of the slabs (RC as well as R/FRC) were controlled by punching shear failures. However, the R/FRC slabs were found to achieve higher shear capacities and failed more gradually than the RC slab. The testing frame used to perform the static and the impact testing is presented in Figure 2.16.

A 300 kg mass with striking velocities ranging from 1.0 to 9.0 m/s was used to impact the specimens at their midpoints. Two impact loading protocols were considered: i) the slabs were progressively loaded to failure under sequential impacts of increasing velocity, or ii) the slabs

were subjected to a single impact event. The authors found that the resulting impact capacities of the slabs were similar from both loading protocols. The authors also concluded that the addition of the PVA short-fibres led to increased shear capacities under the impact loading conditions, reduced concrete scabbing on the bottom surface of the slabs, and the required input energies to fail the slabs under impact were found to increase with the PVA fibre volume fraction.



(a) static testing(b) impact testingFigure 2.16 – Kuriyashi et al. Test Frame (Kuriyashi et al., 2006)

Zhang et al. (2007)

To assess the applicability of using engineered cementitious composite (ECC) panels to construct blast shelters, Zhang et al. performed a series of drop-weight impact tests on ECC, R/FRC, and conventional RC panel/slab specimens. The panels had geometries of 2,000 x 1,000 x 100 mm, and were orthogonally reinforced with 8 mm diameter steel bars spaced at 150 mm on the top and bottom faces of the specimens. The single R/FRC specimen considered in the testing program was constructed with 1 % volume fraction of commercially available end-hooked steel fibres ($l_f = 30$ mm, $d_f = 0.55$ mm, $f_{u,f} = 1,100$ MPa). Line supports were provided on the four sides of the specimens, and a clamping mechanism was used to prevent uplift. Load was applied using a 45 kg impactor with a 95 mm hemispherical striking face, dropped from a height of 4 m. All specimens were subjected to repeated drops until the impactor fully perforated the target.

The ECC panels, which were comprised of both polyethylene and steel fibres, outperformed the R/FRC and the RC panels in terms of energy absorption, local damage behaviour, and improved deformation capabilities prior to failure. However, it is worth noting that with a fibre volume

fraction of 2 % (1.50 % polyethylene, 0.50 % steel), the ECC panels contained a significantly larger total fibre volume fraction than the R/FRC panel. In comparing the results between the R/FRC and RC panels, the addition of the end-hooked steel fibres was also found to reduce the local damage and improve energy absorption capabilities.

2.3 Significance of Current Study

The objective of this chapter was to provide a general overview of research programs which have been carried out in regards to: analyzing RC slab and shell structures under general loading conditions, evaluating the global response of RC slabs and shells under drop-weight impact loading conditions, and assessing the feasibility of using FRC to improve the performance of conventional RC slabs and shells under drop-weight impact.

A significant amount of work has been performed in an effort to develop layered RC shell analysis procedures. The majority of the work presented in the literature can be subdivided into two categories: i) nonlinear finite element programs which are capable of analyzing RC shells and slabs at the structure level but typically simplify, or entirely neglect, out-of-plane shear behaviour, and ii) nonlinear sectional analysis tools which are capable of explicitly computing the out-of-plane shear response, but are only capable of analyzing a single shell element under user-specified sectional forces. Analytical methods which are capable of analyzing RC shells at the structure level and explicitly compute the sectional out-of-plane shear response throughout the structure are currently absent. The development of such complex shell analysis procedures presents major challenges, with perhaps the most significant being that layered finite elements are typically developed on the basis of sectional out-of-plane shear behaviour as opposed to the out-of-plane behaviours of individual layers. Additionally, the computational requirements for such finite element programs would likely make them of limited practical use.

The behaviour of reinforced concrete structures under impact loading conditions is an area of research which is not well understood. The majority of the impact-related investigations carried out have primarily focused on the local impact response of RC elements. As such, the global behaviour of RC shells and slabs under impact represents a research area where limited experimental data exist. The use of layered finite elements to analyze the global impact response of RC shells and slabs under impact has been investigated by some; however, analytical

approaches employed over the last decade have almost entirely been confined to hydrocodes (e.g., *LS-DYNA*) which employ three-dimensional micro-modelling techniques. The use of such analytical methods have been met with limited success as they are demanding in terms of preparation and computation, and as noted by some, require the user to supply modelling parameters which are often difficult to define.

Lastly, the use of modern materials, such as FRC, to improve the performance of conventional RC structures under impact represents a currently emerging area of research. From the limited experimental work which has been performed on this front, R/FRC elements have been shown to exhibit superior performance characteristics under impulsive loading conditions when compared to conventional RC elements. Analytical work pertaining to R/FRC under impact is almost entirely absent from the database of literature.

The study undertaken in this research program is aimed toward the development of an alternative analytical tool which can be used to efficiently analyze RC and R/FRC slab and shell structures under a broad range of loading conditions. Cracked reinforced concrete is modelled in accordance with the Disturbed Stress Field Model (DSFM) (Vecchio, 2000), an extension of the MCFT (Vecchio and Collins, 1986), and FRC is analyzed in accordance with the Simplified Diverse Embedment Model (Lee et al., 2013), a constitutive model developed specifically for steel fibre reinforce concrete elements. Employing advanced constitutive relations, complex shell and slab structures can be modelled using basic finite element modelling techniques and rudimentary material models and input parameters. With the ability to efficiently compute the global impact responses of RC and R/FRC slabs and shells under impact and impulsive loading conditions, the program represents analytical capabilities which are currently absent from the database of literature, particularly with respect to the analysis of R/FRC elements.

In addition to the developed software program, the testing program forming the experimental component of this thesis provides a well-instrumented series of tests in a field of research where only limited data currently exist: the global response of RC and R/FRC slabs under drop-weight impact. It is believed that these data will be useful in the development and appraisal of current and future analytical procedures pertaining to slabs under impact.

CHAPTER 3: EXPERIMENTAL PROGRAM

An experimental testing program consisting of reinforced concrete (RC) and steel fibre reinforced concrete (SFRC) slab specimens subject to drop-weight impact loading was performed. Eight specimens with simple support boundary conditions were tested to failure under sequential high-mass, low-velocity impact loading conditions. The test program was developed to address principal objectives of assessing the potential usage of steel fibres to improve the performance of RC elements under impact loading, and to obtain a 'high-quality' data set pertaining to a research area where limited or no data are available in literature; specifically, data pertaining to SFRC elements coupled with conventional reinforcement under impact conditions (R/FRC elements).

This chapter presents details regarding the constructed slab specimens, the drop-weight test frame, and the developed impact loading protocol. A detailed account of the provided instrumentation and data measurement techniques are presented within this chapter. Lastly, a brief summary of the test methodologies used to characterize the behaviour of the concretes comprising the slab specimens is provided.

3.1 Test Specimens

In total, eight slab specimens of uniform geometry were constructed. Four of the slabs were cast using a conventional concrete mix design, while the remaining four slabs were cast using SFRC mixes with varied steel fibre volume fractions.

3.1.1 Specimen Details

To ensure that the concrete was both consistent and of good quality, especially with regard to the SFRC specimens, the concrete was mixed in-house using a 130 litre pan-mixer located in the University of Toronto's Concrete Materials Laboratory. As such, the overall scale of the specimens was strongly influenced by the volume of concrete which could be mixed and cast within a timely manner. The resulting geometries of the constructed slab specimens were 1,800 mm square with thicknesses of 130 mm.

Two parameters were varied amongst the eight specimens: the fibre volume, V_{f_2} which ranged from zero to 1.50 %, and the longitudinal reinforcement ratios, ρ_x and ρ_y , which were equal in the planar directions and ranged from 0.273 % to 0.592 % per layer. The target 28-day compressive strength for the conventional and the fibre reinforced concrete specimens was 50 MPa, and the maximum nominal aggregate size was 13 mm. A summary of the material composition for the eight slabs, indentified as TH1 through TH8, is presented in Table 3.1.

	1	1				
Slab	Cast Date	Target f_c '	$\rho_{\rm x} = \rho_{\rm y}$	Rebar/Spacing	V_f	
5140	(mm-dd-yy)	(MPa)	$(\%)^{1,2}$	(mm)	(%)	
$\mathrm{TH1}^{*}$	05-17-10	50	0.420	#3 at 130	-	
TH2	06-01-10	50	0.420	#3 at 130	-	
TH3	07-13-10	50	0.420	#3 at 130	0.50	
TH4	07-20-10	50	0.420	#3 at 130	1.00	
TH5	08-17-10	50	0.420	#3 at 130	1.50	
TH6	10-28-10	50	0.273	#3 at 200	-	
TH7	09-28-10	50	0.592	10M at 130	-	
TH8	09-13-10	50	0.592	10M at 130	1.00	

Table 3.1 – Specimen Composition

each layer of steel; each planar direction

² based on total section height (h = 130 mm) * out-of-plane shear reinforcement surrounding impact region

The slabs were designed such that, under monotonic loading conditions, a ductile failure mode consisting of tension steel yielding governed ultimate capacity. The analytical behaviour of the slabs under monotonic loading conditions is presented and discussed for comparative purposes in Chapter 7.

The slabs were doubly reinforced with equal amounts of reinforcement in both planar directions, resulting in four layers of longitudinal steel. The amount of reinforcement in the top layers was equal to that of the bottom layers, and was placed symmetrically through the depths of the slabs. To accommodate the varying reinforcement levels, two types of reinforcing bars were used: U.S. #3 mild bars with a cross-sectional area of 71 mm² and a diameter of 9.5 mm, and CSA Standard No.10 bars with a cross-sectional area of 100 mm² and a diameter of 11.3 mm. The stress-strain behaviour obtained from tensile coupon tests of the reinforcing bars are shown in Figure 3.1. Table 3.2 presents mean properties developed from the test results of the reinforcing bars.



Figure 3.1 - Reinforcing Bar Stress-Strain Behaviour

Table 3.2 –	Reinford	ing Bar	Properties
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Rebar	E_s	f_y	\mathcal{E}_{y}^{1}	\mathcal{E}_{sh}^2	f_u	\mathcal{E}_u^3
	(MPa)	(MPa)	$(x \ 10^{-3})$	$(x \ 10^{-3})$	(MPa)	$(x \ 10^{-3})$
#3	193,000	489	2.54	26.7	597	124
10M	201,700	439	2.19	26.6	564	158

* mean values; based on (3) samples per rebar type

¹ yield strain

² strain at initiation of strain hardening

³ strain corresponding to peak stress, f_u

The rebar spacing within the slabs ranged from 130 mm to 200 mm, which is approximately equivalent to the range of *h* to $1.5 \cdot h$, where *h* represents the overall height of the slab. Note that design provisions commonly prescribe that maximum slab reinforcement spacing not exceed spacing values in the order of $1.5 \cdot h$ in critical locations (CSA A23.3, 2004; ACI, 2008). To ensure sufficient bar development in the support regions, all longitudinal reinforcement bars were detailed with hooked ends. The inner layers of reinforcement were bent with 180° endhooks, and outer reinforcement layers had 135° bends to avoid congestion in the corners of the slabs. Lastly, steel links constructed from bent #3 reinforcing bars were used to join the upper and lower reinforcing mats together, and to prevent movement of the upper mats during casting. The links were placed in the corner regions of the slabs beneath bearing plate locations, and in the slab's midpoint region. In the case of slab TH1, additional links were provided to stiffen the impact region. Figure 3.2 presents the cross-sectional details pertaining to all of the slabs. The overall slab geometries and reinforcement layouts are presented in Figure 3.3 and Figure 3.4.



Figure 3.2 – Slab Cross Section Details (dimensions in millimetres)



Figure 3.3 – Reinforcement Layout for Slab TH1

One fibre type was used in the construction of the SFRC slabs: Dramix[®] RC-80/30-BP high strength, end-hooked steel fibres. These fibres were reported to have the following nominal properties: 30 mm fibre length, diameter of 0.37 mm, aspect ratio of 79.9, and ultimate tensile stress capacity of 2,300 MPa. However, experimental findings showed that the ultimate tensile strength of these fibres were, on average, significantly higher than that reported (Deluce, 2011). The author of the study reported an average ultimate strength of 3,277 MPa for the RC-80/30-BP steel fibres.



3.1.2 Specimen Construction

Two types of concretes were used in the experimental program: a conventional nonfibrous concrete mix, and an SFRC concrete mix incorporating Dramix[®] high strength, hooked-end steel fibres. Both concrete mixes were designed for a target 28-day compressive strength of 50 MPa, and were derived from the mix designs reported by Susetyo (2009). Note that Susetyo's mix designs were originally adapted from a series of 40 MPa mixes reported by Naaman et al., (2006).

Based upon the results from a series of trial mixes, and difficulties encountered in acquiring specific materials, some modifications were made to the mix designs reported by Susetyo. In place of 10 mm (3/8") crushed limestone, 16 mm (5/8") crushed limestone sieved down to 13 mm (1/2") nominal maximum aggregate size was used. To maintain required strength and workability of the concretes, minor deviations from the mix designs reported by Susetyo were required, and pertained primarily to the water content and the chemical admixture dosages. Lastly, because multiple batches from the 130 litre pan-mixer were required to achieve necessary volumes, a retarding admixture was added to the mix designs. Pozzolith 100XR produced by BASF was the retarding admixture used, and it was determined that minimum dosage levels were effective in retarding the mix throughout the full casting process. The dry compositions of the SFRC and the conventional concrete mix designs are provided in Table 3.3.

Material	Unit	Plain	SFRC
type GU cement	kg	375	500
sand	kg	847	1114
water	kg	139	163
13 mm limestone	kg	1080	792
¹ WR: Polyheed 997	mL	1000	2750
² SP: Rheobuild 1000	mL	3500	3500
³ R: Pozzolith 100XR	mL	488	650
0.50% steel fibres	kg	-	39
1.00% steel fibres	kg	-	79
1.50% steel fibres	kg	-	118

Table 3.3 – Concrete Mixture Designs*

* dry composition; per cubic metre of concrete

¹ water reducer

² superplasticizer

³ retarder

Five concrete batches of 115 litres were required for each slab specimen. In addition to casting the slab specimen, concrete cylinders, concrete bending prisms, and concrete 'dog-bone' specimens were cast to characterize the behavioural properties of the concrete material. The purpose and methodology pertaining to the companion specimen testing is described in detail in Section 3.6.

The same mixing procedure was used for both the SFRC and the conventional concrete mixes, and was primarily based on that presented by Susetyo (2009). The procedure used was as follows:

- 1. The coarse aggregate, sand, and cement were dry-mixed for three minutes.
- 2. The water reducer (Polyheed 997) and one-half of the required water were added and allowed to mix for three additional minutes.
- 3. The mixing process was paused for two minutes.
- 4. Three-quarters of the remaining water and the full amount of superplasticizer (Rheobuild 1000) were added; mixing continued for one minute.
- 5. The remaining water and the retarder (Pozzolith 100XR) were added, and the concrete was allowed to mix for another two minutes.
- 6. (*SFRC*): The steel fibres were slowly incorporated into the concrete as mixing continued. Mixing was stopped when all fibres were added and appeared to be dispersed without the presence of fibre balling or clumping.
- 7. The concrete was loaded into a buggy and transported for casting. A light cleaning of the mixer was performed prior to loading materials and repeating the process for the following batch.

The concrete was shoveled from the buggy into a wooden form constructed for the slab specimens. The formwork was mounted on an elevated mechanical vibrating table (see Figure 3.5). The concrete was placed such that the casting process commenced in one corner of the slab, and was completed with placement of concrete in the diagonally opposite corner. This placement method minimized the possibility of cold-joint development. The concrete was externally vibrated for approximately 10 to 15 seconds between each batch of concrete. Once all required concrete had been placed and vibrated, the slab surface was finished with a trowel to produce a

smooth, void-free surface (see Figure 3.6). The slabs were tented under plastic and allowed to cure for a period of three days under moistened burlap. After the three-day cure period, the slabs were stripped from their formwork and stored until testing.



Figure 3.5 – Mechanical Vibrating Table



Figure 3.6 – Finished Slab Specimen

In addition to the concrete slab specimens, several companion specimens were cast as a means of evaluating the concrete material properties, and to further investigate material behaviour. From

each batch of concrete used in the construction of the slab specimen, two standard 150 x 300 mm (6" x 12") cylinders were cast. In total, three concrete bending prisms were cast, from different concrete batches, for each slab. The concrete for the cylinders and the bending prisms was placed in three lifts, and was vibrated using an external vibrating table in accordance with ASTM Standard C192/C192M. Additionally, a minimum of two 'dog-bone' specimens were cast for each SFRC slab specimen. The material for the 'dog-bones' was placed in two lifts, and externally vibrated after the each lift.

All companion specimens were demoulded after one day and match-cured with the slab until testing. After the three day moist-curing period, one cylinder from each batch of concrete was transferred to a moist-curing room and the remaining cylinders were left to cure alongside the slab. Specimens stored in the moist-curing room were used to evaluate the 28-day compressive strength of the concrete.

3.2 Test Frame

All of the slab specimens in this study were restrained at their corners, and impacted at their midpoints. The decision to restrain the slabs at the corners in lieu of the more common approach where continuous vertical support is provided along the slab edges was made for several reasons: under a concentrated impact condition, axial expansions which occur non-uniformly along the perimeter of the slab would require that multiple independent roller supports be used to provide the vertical restraint along a single edge of the slab; due to the square geometries of the slabs, the forces developed in the vertical support reactions will also be non-uniform, with corner regions having the tendency to uplift under positive flexure; and the use of corner supports reduces the measurement of the reaction forces to discrete locations pertaining to only the corners of the slab.

Previous studies presented in the literature have shown that structures under impulsive loading conditions often display significant displacement rebounding leading to specimen uplift (Ho, 2004). As such, a restraint methodology similar to that used in previous impact loading experiments performed at the University of Toronto (Saatci and Vecchio, 2009) was employed. At the four corner supports, vertical translations were restrained in both the positive and negative directions but with the supports free to rotate. Lateral translational restraint was varied amongst

the four corners in an effort to minimize axial confinement effects but still provide a stable support condition.

Details of the support assembly are provided in Figure 3.7. On the top surface of the slab, spherical ball bearings reacting against stiff HSS sections were used to prevent slab uplift but allow the slab to translate laterally and rotate freely. The HSS sections were anchored to the base support columns using high-strength Dywidag rods, which were anchored to a concrete strong floor. The tie-down assembly was post-tensioned such that high tension forces were developed within the tie-down reaction assembly and between the Dywidag nuts anchoring the HSS section, but only negligible tensile force (approximately 5 kN per rod) was developed between the base of the HSS section and the top of the tie-down reaction assembly. This post-tensioning method ensured that even under shock and high frequency vibration, all of the Dywidag nuts remained tight throughout the duration of the tests, and that the minimal forces transferred from the tie-down assembly to the slab specimen would provide negligible rotational restraint.



Figure 3.7 – Test Frame Details; East Support (dimensions in millimetres)

To ensure that the slab remained stable during testing, the lateral restraints beneath the specimen were varied amongst the four support locations (see Figure 3.8). The boundary conditions of the East support was that of a simple pin, the West support was free to translate laterally in all directions, and the North and South supports were allowed to translate laterally in only one direction. It was felt that this restraint condition would minimize the development of axial confinement while still maintaining overall stability. The varied support assemblies used in the test frame are illustrated in the elevation presented in Figure 3.9.

3.3 Drop-Weight

The mass of the drop-weight used in the experimental program ranged from 120 kg to 300 kg. The drop-weight was constructed from a 300 mm square HSS section filled with concrete. Steel plates welded to the top and bottom of the section housed a guide tube constructed from Sch. 80 PVC pipe which was oriented vertically through the centre of the drop-weight. The striking surface of the drop-weight was 300 x 300 mm with a flat contact face. Additional steel plates welded to the sides of the drop-weight served as mounting plates, allowing additional mass to be fastened to the drop-weight as required (see Figure 3.10).

The impact loads were generated by what was essentially a free-fall condition of the dropweight. A single vertical rail comprised of Sch. 60 steel pipe with a 64 mm (2-1/2") diameter ran through the centre of the drop-weight and was used to guide the weight during the fall. The guide rail assembly was mounted to a steel frame consisting of two columns and a cross-beam which spanned approximately 4.5 metres above the test slab. To prevent possible damage to sensors located on the top surface of the slab, the bottom end of the rail was positioned 150 mm from the top face of the slab, preventing the weight from deviating significantly from the desired impact region. Immediately prior to performing a test, the drop-weight was lifted up the guide rail to the desired height using an electric chain winch. The drop-weight was then transferred from the winch and secured to a simple clamping style release mechanism. The mass was ultimately released using a fulcrum-based hand lever (see Figure 3.11). Figure 3.12 shows the drop-weight positioned immediately prior to performing a drop-test.



Figure 3.8 – Test Frame; Plan View (dimensions in millimetres)



Figure 3.9 – Test Frame; South Elevation



Figure 3.10 – Drop-Weight (270 kg)



Figure 3.11 – Weight Release Mechanism



Figure 3.12 – Weight Prior To Drop-Test

3.4 Instrumentation

To fully document the behaviours exhibited by the test slabs, a detailed instrumentation scheme was used to acquire specimen displacements, accelerations, support reactions, reinforcing bar strains, and applied impact loads. The following section of the thesis presents details pertaining to the instrumentation used throughout the testing program. Where appropriate, typical instrumentation plans have been provided. A more complete summary of the instrumentation details pertaining to each specimen is provided in Appendix A.

3.4.2 Accelerometers

In total, thirteen accelerometers were used in each test. Eleven accelerometers were used to measure the transverse acceleration behaviour of the slab specimens, and two accelerometers mounted to the drop-weight were used to estimate the applied impact force. The instruments were capable of measuring accelerations along a single axis of motion. Accelerometers capable of measuring accelerations over the range of \pm 10,000 g and \pm 2,500 g were used.

Although some allowances were made for deviations from perfectly symmetric support conditions, it was assumed that the out-of-plane accelerations of the slabs would exhibit similar responses in all four quadrants of the slab. As such, one-quarter of the slab (the South quadrant) was densely instrumented with eight of the eleven accelerometers. The remaining quadrants each contained a single accelerometer for the purpose of verifying the above assumption. The accelerometer instrumentation plan which was used for test slabs TH2 through TH8 is provided in Figure 3.13.

The accelerometer mounting assemblies consisted of aluminum mounting blocks which housed the accelerometers. The mounting blocks were fastened to stiff aluminum anchor plates, which were mounted to the surface of the slab using self-tapping concrete screws. The accelerometer slab mounting assembly is presented in Figure 3.14. With the exception of one accelerometer provided within the mass impact region, all of the accelerometers were mounted to the top surface of the slab using procedure described above. The accelerometer on the bottom surface of the slab was mounted using an anchor plate which threaded directly into a steel coupler cast within the slab. The steel coupler was welded to the bottom mat of reinforcement.



Figure 3.13 – Typical Accelerometer Instrumentation Plan (dimensions in millimetres)



Figure 3.14 – Accelerometer Mounting Assembly

Two accelerometers were mounted to the drop-weight as a means of calculating the applied impact force. Aluminum mounting blocks, which housed the accelerometers, were fastened to opposite faces of the HSS section forming the drop-weight. During the initial pilot drop-tests, it was found that the drop-weight accelerometers routinely saturated due to high frequency vibrations occurring within the steel mass. To minimize the saturation and capture meaningful data from the sensors, a 2 mm thick neoprene pad was compressed between the aluminum mounting blocks and the steel HSS section forming the drop-weight. The accelerometers mounted to the drop-weight are shown in Figure 3.15.



Figure 3.15 - Accelerometer Mounted to Drop-Weight

3.4.3 Load Cells

Load cells were provided at each of the four corner supports to measure the positive vertical support reactions (positive referring to a reaction force directed upward). The load cells were manufactured at the University of Toronto with a static load capacity of 200 kN. The load cells were placed beneath spherical pins provided at each of the supports, between a pair of 40 mm steel plates (see Figure 3.16). The load cells were calibrated under static loads up to their full capacities prior to performing the drop-tests.



Figure 3.16 – Load Cell Configuration

3.4.4 Potentiometers

The slabs were instrumented with a series of potentiometers to measure transverse and lateral slab displacements. Potentiometers are typically capable of capturing higher response frequencies than LVDTs, making them better suited for impact testing applications. This was illustrated in a previous impact testing program performed at the University of Toronto (Saatci, 2007). Two types of potentiometers were primarily used in this program: Duncan Electronics Models 601 and 602 with displacement ranges of \pm 25 mm and \pm 50 mm, respectively. One potentiometer produced by BEI sensors with a displacement range of \pm 75 mm was also used.

Up to twenty-five potentiometers, identified as P1 through P25, were used to measure the displacement behaviours of the slabs. One potentiometer was placed on each side face of the slab

to measure lateral movement. All other potentiometers were used to measure transverse displacements. The potentiometers were placed such that the midspan profiles of the slab were measured across each planar direction. Additional potentiometers were placed at the centre-points of the slab quadrants. After completing the testing of pilot slab TH1, the addition of sensor P25 and the repositioning of P18 (renamed P26) allowed for the measurement of possible transverse movement of the East support assembly. It was assumed that measured movement of the East support could serve as a representative value for all supports. The typical potentiometer instrumentation plan which was used for the majority of the drop tests is presented in Figure 3.17. The potentiometer instrumentation plan used for each slab is provided in Appendix A.



Figure 3.17 – Typical Potentiometer Instrumentation Plan (dimensions in millimetres)

The potentiometers were mounted beneath the slab, using a series of magnetic bases fixed to an anchored steel plate. Plywood sheeting was used to house and protect the sensors from concrete scabbing and debris. Aluminum extension rods were used to connect the potentiometers to segments of threaded rod doweled into the base of the slab. Initially, shrink tubing was used to connect the extension rods to the potentiometers and the anchored dowels, but the shrink tubing

regularly ruptured under the impulsive behaviour. As a result, threaded brass couplers were ultimately used in place of the shrink tubing (see Figure 3.18).



(a) Connection to Potentiometer(b) Connection to Threaded DowelFigure 3.18 – Potentiometer Mounting Assembly

During slab testing, a number of potentiometer measurements had to be discarded as the functionality of the instrument was disrupted. These measurement errors occurred for various reasons: internal sensor damage due to shock, dislodging of a dowel from the slab, or ejection of the concrete supporting a threaded dowel. In an effort to ensure that at least the midpoint displacement measurement was captured in every test, two potentiometers were used at the midpoint location. Figure 3.19 shows the lateral and transverse potentiometer placement methods.

3.4.5 Strain Gauges

Strain gauges were used to achieve two goals in this experimental program: to determine a general range for the magnitude and rate of strain in the reinforcing bars within the slab specimens, and to measure the strains in the Dywidag bars forming the vertical tie-down assemblies of the corner supports. For both applications, TML FLA-5-11 strain gauges with 5 mm gauge lengths were used. The gauges were manufactured by Tokyo Sokki Kenkyujo Co. Ltd.



Figure 3.19 – Potentiometer Placement

Sixteen strain gauges were applied to the Dywidag bars used in the tie-down assemblies of corner supports. Each of the eight Dywidag bars was instrumented with two strain gauges, placed on opposite faces of the bar (see Figure 3.20). The average value from the two strain gauge measurements was used to calculate the negative support reaction (negative referring to a reaction force directed downward) at each of the corner supports.



Figure 3.20 – Strain Gauged Tie-Down Assembly

Up to twelve strain gauges were applied to the reinforcing bars within each slab specimen. The strain gauges were focused within, and around, the midpoint impact region of the slabs, and were distributed throughout all four layers of reinforcement. The typical strain gauge instrumentation plan which was used for most of the test slabs is presented in Figure 3.21. The strain gauge instrumentation plan used for each slab is provided in Appendix A.



Figure 3.21 – Typical Strain Gauge Instrumentation Plan (dimensions in millimetres)

3.4.6 High-Speed Video

In addition to a standard imaging hand-held video camera, a high-speed video camera was used to capture additional details regarding the drop-weight impact. The camera was positioned to capture the southwest elevation of the test slab, and approximately a one metre length of the drop-weight guide rail suspended above the slab. At a recording rate of 3,000 frames per second, the camera was used to verify the impact velocity of the drop-weight. However, because of the high frame rate that was used during the tests, the video resolution and the recording duration were limited.

3.4.7 Data Collection System

Two data acquisition systems, both of which were produced by Hottinger Baldwin Messtechnik GmbH (HBM), were used to acquire digital data during the impact tests: HBM's QuantumX system, and HBM's MGCplus system.

The QuantumX system was developed for applications requiring data sampled at high rates. The system is compatible with various sensor types, and is capable of acquiring unfiltered data at sampling rates ranging from 2,400 Hz to 192,000 Hz. The use of such high acquisition rates are necessary to capture high frequency behaviours exhibited by structures under impact, or impulsive type, loading conditions. Due to the extreme volume of data that are collected over a short period of time, the number of channels and the duration of the event from which data can be simultaneously sampled are limited. In this study, the QuantumX system was used to capture data from instruments which were expected to possess the highest frequency content: the accelerometers and the load cells.

The MGCplus data acquisition system is a modular system comprised of multiple connection boards which are linked for simultaneous acquisition. The system was configured such that data from many types of instruments could be acquired. The maximum sampling rate available using the system assembly was 2,400 Hz; however, it was possible to collect data from a few channels at an increased sampling rate of 19,200 Hz. Instrument data collected using the MGCplus system included potentiometers, strain gauges mounted to the support assemblies, and the strain gauges mounted to the slab reinforcement.

It should be noted that because two acquisition systems were used to collect digital data, the acquired data sets were not time synchronized. As a means of realigning the data sets, post-test, the data from a single transverse potentiometer mounted to the slab specimen was acquired using both acquisition systems. The alignment potentiometer was sampled at a rate of 96,000 Hz using the QuantumX system, and at a rate of 19,200 Hz using the MGCplus system. A summary of the sampling rates and acquisition systems used for each instrument has been provided in Table 3.4.
Songor Tymo	Name System / Board		#	Sampling
Sensor Type	Inallie	System / Board	Channels	Rate
Load Cell	LC1–LC4	QuantumX	4	96,000 Hz
Potentiometer	P1-P19 ; P21-P25	MGCplus / AP836	24	2,400 Hz
Potentiometer	$P20^*$	QuantumX	1	96,000 Hz
Potentiometer	$P20^*$	MGCplus / AP01i	1	19,200 Hz
Strain Gauge (reinforcement)	S1-S12	MGCplus / AP801	12	2,400 Hz
Strain Gauge (Dywidag)	(see Figure 3.21)	MGCplus / AP801	16	2,400 Hz
Accelerometer (\pm 10,000 g)	A1 ; A13	QuantumX	2	96,000 Hz
Accelerometer ($\pm 2,500$ g)	A2-A11	QuantumX	11	96,000 Hz

Table 3.4 – Data Acquisition Summary

* used to synchronize data, post-test

3.5 Loading Protocol

The slabs in the experimental program were tested under sequential impacts of increasing mass, with constant impact velocities. The drop-height of the impact mass was set at 3.26 m above the top surface of the slab, resulting in a nominal impact velocity of 8.0 m/s. It should be noted that in a few impact events, slight changes in the drop-height occurred due to modification of the impacting mass and, in some cases, large permanent residual deflections of the slabs. However, the high-speed video footage verified that little deviation from the desired nominal velocity occurred over the course of the testing program.

For all but one of the test slabs, pilot slab TH1, a single loading protocol was used to perform the drop-weight testing. The typical loading protocol, which was used for slabs TH2 through TH8, consisted of an initial impacting mass of 150 kg. For subsequent impacts, the mass level was increased by 30 kg increments until an impacting mass of 240 kg was achieved (the fourth impact). To ensure that the slabs, which typically displayed significant damage after subjection to four impacts, could safely withstand impacts of increased mass levels, repeated impacts were performed for the 240 kg mass level and the 270 kg mass level (impact events 4 through 7). After the performance of seven impact events, a 300 kg drop-mass was used for the remainder of the drop-tests (tests 8 through 10). For slab TH1, three drop-tests were performed with an initial impact event consisting of a 120 kg drop-mass. Based upon the behaviour exhibited by slab TH1 under the performed loading protocol used for the drop-weight testing is presented in Table 3.5.

Three criteria were considered in terminating the testing of slabs TH2 through TH8: i) the impact loading protocol was completed (see Table 3.5), ii) the measured support reaction forces decreased significantly from those measured during the previous impact event, or iii) the damaged state of the slab increased the likelihood of the instrumentation being damaged under the performance of an additional impact event. The occurrence of any one of the above criteria constituted completion of the slab impact testing.

14010 5.5	Louding 1 lotocol				
- "	Drop-Weight (kg)				
Impact #	TH1	TH2–TH8			
1	120	150			
2	150	180			
3	180	210			
4	-	240			
5	-	240			
6	-	270			
7	-	270			
8	-	300			
9	-	300			
10	-	300			

Table 3.5 – Loading Protocol

3.6 Companion Specimens

A series of companion specimens were cast alongside the slabs for the purpose of evaluating the mechanical properties of the concrete. The companion specimens were in the form of standard cylinders which were used to evaluate the concrete compressive properties; beam prisms, used to determine the flexural tensile strength of the concrete and, in the case of SFRC, the post-peak bending characteristics of the concrete; and uniaxial tension specimens, which were used to evaluate the tensile behaviour of the concrete used in the construction of the SFRC slab specimens. Upon completion of the drop-weight testing, a series of concrete cores was cut from the SFRC slabs. The core samples were used to evaluate the in-situ characteristics of the fibres within the concrete. Results from the companion specimen tests are provided in Chapter 4.

3.6.1 Concrete Compressive Strength Tests

For each slab specimen, ten standard size concrete cylinders (150 x 300 mm) were cast and tested to evaluate the properties of the concrete under axial compression. Five moist-cured cylinders were tested on the 28^{th} day of curing, and were used primarily for the purpose of

evaluating the compressive strength of the concrete mix design. Five match-cured cylinders were tested at the same time as the slab specimens and were used to estimate the compressive properties of the slabs at the time of testing.

Prior to testing the concrete cylinders, the cylinder ends were ground smooth to remove surface paste and to reduce the occurrence of stress concentrations resulting from surface defects. In the case of SFRC specimens, the cylinder ends were saw-cut and end-capped to prevent stress concentrations resulting from surface irregularities caused by steel fibres. The cylinders were loaded to failure in an MTS testing machine with a load capacity of 4,500 kN. Testing was performed in a displacement controlled manner at a rate of 0.00667 mm/s. Match-cured cylinders were instrumented using two \pm 2.5 mm LVDTs, mounted over a gauge-length of 250 mm, for the purpose of measuring the axial strains of the cylinders (see Figure 3.22).



Figure 3.22 – Instrumented Fibre Reinforced Concrete Cylinder

3.6.2 Concrete Bending Tests

A minimum of two concrete prisms corresponding to each slab specimen were cast for the purpose of performing concrete bending tests. The bending tests were used to evaluate the flexural tensile strength of the concrete and, in the case of SFRC, evaluate post-peak flexural characteristics exhibited by the concrete. Prisms were cast from different concrete batches used in the construction of each slab and were match-cured alongside the slab specimens. The prisms were tested after completion of the slab impact testing.

All bending prisms were of uniform geometry (152 x 152 x 533 mm) and were tested under third-point loading in adherence with the ASTM C1609 standardized test method (see Figure 3.23). The specimens were loaded in a displacement-controlled manner at a machine displacement rate of 0.0060 mm/s. An instrumentation configuration proposed by the RILEM Technical Committee (RILEM TC162, 2002) was used to acquire data for the concrete bending tests. Two LVDTs were used to measure the midspan displacements. The LVDT's were mounted to aluminum bars which were clamped to each face of the prism at mid-depth, over the support location. LVDTs were positioned relative to the supports, and reacted against aluminum plates mounted to the specimen at the midspan. This method of displacement measurement ensured that support movement, and localized deformation under the loading points, was not erroneously measured as displacements resulting from prism bending (see Figure 3.24).



Figure 3.23 – Instrumented Concrete Prism



Figure 3.24 – Test Frame for Concrete Prisms (dimensions in millimetres)

3.6.3 SFRC Uniaxial Tension Tests

For each SFRC slab, a minimum of two concrete 'dog-bone' specimens were constructed for the purpose of performing uniaxial tension tests. Owing to their identifiable shape, the specimens referred to as 'dog-bones' were used to evaluate the direct tensile performance of the concrete material comprising each of the SFRC slabs.

The 'dog-bone' testing method is an adaptation of that proposed by the RILEM TC162 (2001). The method proposed by the RILEM committee involves the use of a notched cylinder under axial tension. As a result of the fabricated notch, cracking is forced to occur within a predefined weakened region of the specimen, and deformations are measured explicitly across the notch. In contrast, the 'dog-bone' testing method used in this study measures deformations over an extended gauge length, allowing cracks to form freely, and in some cases capturing the formation of multiple cracks within the gauge length.

The 'dog-bones' were cast and cured alongside the slabs until testing. Each 'dog-bone' was constructed using material from a single concrete batch. It was assumed that a minimum of two 'dog-bones', representing two of the five batches of concrete required to cast a single slab, would serve as a representative sample of the material comprising each slab. Prior to testing, the ends of the specimens were ground down to remove weak surface paste and expose the coarse aggregate. With the use of a diamond cutting wheel, striations approximately 3 mm in depth were cut into the end surfaces. Lastly, steel end blocks were fastened to the ends of the 'dog-bones' using Sikadur30TM high strength epoxy, and were allowed to cure for a minimum of three days before testing (see Figure 3.25).

The 'dog-bones' were constructed 500 mm in length with thicknesses of 70 mm. Within the effective gauge length, the cross-sectional area of the specimens was reduced, resulting in cross-sectional dimensions of 100 x 70 mm. Four LVDTs were used to measure the axial deformations of the specimen. Two LVDTs were placed on the sides of the 'dog-bones' within the reduced cross-section and had effective gauge lengths of 150 mm. The other two LVDTs were placed on opposite faces of the specimen with an increased gauge length of 300 mm. The geometry of the 'dog-bones', and the placement of the LVDTs is presented in Figure 3.26.



Figure 3.25 - 'Dog-Bone' End Block Mounting

The 'dog-bones' were tested using an MTS universal testing machine with a capacity of 245 kN. The specimens were fastened to rotating ball-joints which threaded into the end blocks mounted to the 'dog-bones,' resulting in pinned end conditions. The test was performed in a closed-loop, displacement controlled method. The displacement rate was controlled using LVDTs mounted to the test frame which measured displacements between the upper and lower pin supports. Ideally, the displacement rate of the test should be controlled by LVDTs mounted directly on the specimen; however, previous studies using this method resulted in little success (Susetyo, 2009).



Figure 3.26 - 'Dog-Bone' Specimen Details (Deluce, 2011) (dimensions in millimetres)

The test was performed using an initial displacement rate of 0.001 mm/s. Once the peak load level had been obtained, and significant softening of the load-deformation behaviour was observed, the displacement rate was incrementally increased up to a maximum value of 0.010 mm/s. Data were acquired at a rate of 20 Hz throughout the duration of the test to ensure that any abrupt changes in the measured load were adequately captured. Figure 3.27 shows a 'dog-bone' specimen mounted in the testing frame.



Figure 3.27 - 'Dog-Bone' Test Setup

3.6.4 In-Situ SFRC Composition

Upon completion of the drop-weight testing, a series of cores were sampled from several of the SFRC slabs. The cores were used to evaluate the dispersion and orientation of the steel fibres within the test slabs. The sampled cores had diameters of either 100 mm or 150 mm. In most cases, six cores were taken from each slab sampled (see Figure 3.28).

Each of the sampled cores was subdivided into six sections. Using a concrete target saw, two cuts were made over the height of the cores, and a single vertical cut was made through the cores

along the planar direction of the slab. The exposed fibres were counted and documented for each cross-section revealed.



Figure 3.28 – Core Sampling From SFRC Slab

In addition to the fibre data collected from the core samples, the 'dog-bone' specimens were also used to evaluate concrete fibre composition. Upon completion of the uniaxial testing, the 'dog-bones' were pulled apart into two pieces using the test frame presented in Section 3.6.3. The 'dog-bones' were removed from the testing machine, and the exposed fibres crossing the plane of separation were counted and documented.

CHAPTER 4: TEST RESULTS AND DISCUSSION

An experimental program consisting of RC and R/FRC slab specimens subject to drop-weight impact loading was performed. The test program, which has been summarized in the previous chapter, was developed to assess the suitability of using steel fibres for the purpose of impact resistant design, and to obtain a well-documented data set which could be used to corroborate and further develop numerical analysis methods.

This chapter presents the experimental results from the test program undertaken. The chapter begins with the presentation of the results from the companion specimens, which were tabulated and used to characterize the mechanical properties of the concrete comprising the slabs. Additional discussions pertaining to the observed influence of the steel fibres in the companion specimens is provided.

A detailed account of the test observations, and the visible damage progression resulting from repeated impact loading, is provided for each slab specimen. Key results from the collected digital data set are presented. A brief discussion pertaining to the validity of the collected data from the various sensor types and acquisition systems used in the program is provided. Discussion pertaining to trends exhibited by the slab specimens and the influence of the steel fibres is presented.

4.1 Companion Specimen Test Results

Key results obtained from the companion specimen tests are provided in the following section. Mechanical properties pertaining to each parent test slab are summarized in tabular form. A brief discussion pertaining to the influence of the steel fibres in each of the companion studies is provided. A comprehensive listing of all of the companion specimen test results is presented in Appendix B.

4.1.1 Cylinder Compression Tests

A series of ten concrete cylinders were tested for each slab specimen. The tests were performed to evaluate material properties which are commonly used to characterize the compressive properties of concrete: the compressive strength (at 28 days and at the time of slab testing), the secant modulus of elasticity, and the compressive strain developed under the peak stress. Table 4.1 presents a summary of the cylinder results.

Parent Slab	Cylinder Test Age ¹ (days)	V _f (%)	<i>fc</i> ' 28 day (MPa)	f_c ' test age (MPa)	\mathcal{E}_c^{\prime} test age (x 10 ⁻³ mm/mm)	E_{cs}^{2} test age (MPa)
TH1	284	-	54.0	74.1	2.490	40,700
TH2	318	-	58.0	69.4	2.554	36,800
TH3	304	0.50	39.9	48.0	2.802	28,800
TH4	340	1.00	41.3	48.6	3.296	25,600
TH5	340	1.50	46.2	50.7	4.188	23,300
TH6	284	-	55.5	59.0	2.670	32,000
TH7	319	-	54.9	60.3	2.635	34,600
TH8	353	1.00	42.8	45.1	3.367	24,000

Table 4.1 – Concrete Compressive Properties

¹ at time of slab impact testing

² in accordance with ASTM C 469 (2002)

The values in Table 4.1 represent mean values obtained from testing five cylinders at 28 days, and five cylinders at the time of slab testing. The five cylinders were sampled from all five batches of concrete comprising each slab. It should be noted that the cylinder results presented for TH1 are based on a reduced data set of only three cylinders. The removal of two outlying cylinders, presented in Appendix B, was deemed appropriate based on compressive test results obtained from core samples taken from slab TH1 (see Figure 4.1). An average compressive strength of 54 MPa was obtained from five cores sampled from TH1 after completion of the slab impact protocol, verifying that the outlying cylinder results were not representative of the concrete within the slab. The cores were 95 mm in diameter, and were sampled and tested in accordance with ASTM C 42 (2003). Minor strength corrections were performed on the compressive results to account for core geometry (Bartlett and MacGregor, 1994).

As shown in Table 4.1, the compressive strength and stiffness values of the plain concrete cylinders were significantly higher than those obtained for the SFRC cylinders. Additionally, as the fibre volume fraction increased, the compressive strain coinciding with the peak stress also increased significantly. The fibre volume fraction was found to have little effect on the compressive strength of the concrete; however, the secant stiffness values of the SFRCs decreased as fibre content increased.



(a) core sampling (b) core testing Figure 4.1 – TH1 Post-Test Core Samples

The most apparent difference between the behaviours of the plain concretes and the SFRCs was observed in the measured post-peak responses. In the case of the plain concretes, brittle crushing followed by a rapid strength loss characterized failure. In contrast, SFRC cylinders possessed ductile post-peak behaviours with stable and gradual strength decay. The SFRCs were able to maintain considerable loads at large strain levels. The compressive stress versus strain behaviours from selected cylinder tests have been presented in Figure 4.2. The normalized stress versus strain behaviour of the cylinders has also been presented for comparative purposes. It should be noted that testing of instrumented plain concrete cylinders was terminated immediately after attaining peak load to prevent LVDT damage.

As illustrated in Figure 4.2, the fibre volume fraction had a significant effect on the rate of postpeak strength decay. As the fibre volume fraction increased, the concrete's ability to sustain load in the post-peak regime increased considerably. At strain levels as large as three times that recorded at the peak compressive stress ($\varepsilon_c = 3 \cdot \varepsilon'_c$), the concrete with a fibre volume fraction of 1.50 % was able to maintain approximately 80 % of its ultimate capacity and the SFRC with a fibre volume of 0.50 % maintained approximately 60 % of its ultimate capacity. Conventional concretes typically exhibit negligible residual stresses beyond compressive strains greater than two times that obtained at peak stress ($\varepsilon_c = 2 \cdot \varepsilon'_c$). As illustrated in Figure 4.2, the addition of the relatively small volumes of the steel fibres used in this study significantly impacted the post-peak compressive responses of the concrete.



Figure 4.2 - Concrete Compressive Stress versus Strain Behaviour

The ductile behaviour of the SFRCs was also evident from the damage attained and the cracking behaviour observed during cylinder testing (see Figure 4.3). Plain concrete cylinders crushed in a brittle manner, immediately after achieving their ultimate capacities. After initial crushing, the SFRC cylinders remained intact with the steel fibres preventing the concrete from fracturing.



(a) $V_f = 0$ (TH2) (b) $V_f = 0.50$ % (TH3) (c) $V_f = 1.00$ % (TH4) (d) $V_f = 1.50$ % (TH5) Figure 4.3 – Cylinder Test Photos

4.1.2 Concrete Prism Testing

Bending prisms are commonly tested to evaluate a concrete's flexural tensile strength. Because the specimens are basic in form, and the test is relatively simple to perform, this indirect tensile test is often preferred over direct tension methods for the purpose of estimating tensile strength. In this experimental program, two to three concrete bending prisms were tested to evaluate the flexural tensile strength of the concrete forming each slab. Additional characteristics, specifically with regard to SFRC prisms, were obtained from the bending tests and were used to compare the behaviours of the concretes comprising the slab specimens. The prisms were tested after impact loading of the parent slab specimen was completed.

In general, the addition of steel fibres resulted in significant increases in the ductility exhibited by the prisms. Plain concrete prisms failed immediately after initial crack formation in a brittle manner. In contrast, SFRC specimens displayed gradual post-peak strength decay, often achieved peak flexural stresses greater than the cracking stresses, and developed multiple distributed cracks. The cracking patterns obtained from prisms containing the different fibre volume fractions considered in this program are presented in Figure 4.4. The average peak flexural strength pertaining to each slab is presented in Table 4.2.



(a) $V_f = 0$ (TH2)

(c) $V_f = 1.00 \%$ (TH4)

TH4 B4 JUNE 07/11





(d) $V_f = 1.50 \%$ (TH5)



Parent Slab (# of prisms)	Prism Test Age ¹ (days)	V _f (%)	f_c , ² (MPa)	f_{cr}^{3} (MPa)	f_r^4 (MPa)	$\begin{array}{c} f_t^{*5}\\ \text{(MPa)} \end{array}$
TH1 (2)	380	-	74.1	8.38	8.38	4.61
TH2 (3)	365	-	69.4	8.51	8.51	4.68
TH3 (3)	321	0.50	48.0	6.87	6.90	3.79
TH4 (3)	317	1.00	48.6	6.63	7.98	4.39
TH5 (2)	386	1.50	50.7	6.83	8.76	4.82
TH6 (3)	324	-	59.0	7.92	7.92	4.35
TH7 (3)	345	-	60.3	8.09	8.09	4.45
TH8 (3)	360	1.00	45.1	5.68	6.14	3.37

Table 4.2 – Flexural Tensile Strengths

¹ prism tests were performed after slab impact testing

² results from cylinder compression testing (see Section 4.1.1)
³ flexural tensile strength at first crack

⁴ peak flexural tensile strength

⁵ estimate of direct tensile strength: $f_t^* \cong 0.55 \cdot f_r$ (note: $f_t^* / f_r \cong 0.33 \sqrt{f_c'} / 0.60 \sqrt{f_c'}$)

Flexural tensile stress values were calculated at the first occurrence of cracking and at the peak flexural resistance in accordance with ASTM C 1609:

$$f_{cr} = \frac{P_{cr}L}{bd^2} \quad ; \quad f_r = \frac{P_uL}{bd^2} \tag{4-1}$$

where,

b = prism width, mm;

d = prism depth, mm;

L = clear span of the prism, mm;

 f_{cr} = stress in the extreme tensile fibres at first crack, MPa;

 f_r = stress in the extreme tensile fibres at peak load, MPa;

 $P_{cr} = \text{load}$ (total) at first cracking, N;

 P_u = peak load (total), N.

Note that in Equation 4-1, it is inherently assumed that the neutral axis of the prisms is located at mid-depth and that the stress distribution of the concrete within the flexural region remains linearly distributed through the depth of the specimen. However, after flexural cracking occurs, the above stated assumptions are no longer valid. As such, the calculated ultimate flexural tensile strength, f_r , values reported in Table 4.2, should be considered representative sectional properties as opposed to an estimate of the actual tensile stresses occurring within the prisms.

The addition of the steel fibres typically resulted in increased flexural strengths and improved ductility. Prisms containing fibre volume fractions of 1.00 % or more exhibited stable strain hardening behaviours which were characterized by flexural resistances exceeding those measured at initial cracking. In addition to the increased flexural capacities, the occurrence of strain hardening was also evident from distributed crack development (see Figure 4.5). Prisms containing fibre volume fractions of 0.50 % exhibited a more rapid decay of the flexural resistance and, on average; their flexural capacities were governed by the initial crack formation. Note that, for clarity, photographed cracks in Figure 4.5 have been magnified.



Figure 4.5 – Bending Prism Multiple Cracking; $V_f = 1.50 \%$ (TH5)

Figure 4.6 presents selected load versus deflection behaviours obtained from the prism testing. The figure illustrates the increases in strength and ductility attained as a result of increasing fibre content. The load versus deflection behaviour for all of the instrumented prisms has been presented in Appendix B.

4.1.3 SFRC 'Dog-Bone' Tests

A series of concrete 'dog-bone' specimens were tested to evaluate the behaviour of SFRC under direct tensile loading. The tests were used to determine characteristic tensile properties of the SFRC forming the slabs, evaluate the post-peak behaviour of SFRC under direct tension, and to corroborate the flexural tension results obtained from the concrete bending tests presented in Section 4.1.2.



Figure 4.6 – Typical Load versus Deflection Behaviour for Concrete Prisms

For each R/FRC slab, either two or three 'dog-bone' specimens were used to represent the concrete material comprising the parent slabs. Results from the 'dog-bone' tests were used to determine averaged characteristic values, and are presented in Table 4.3.

Parent Slab (# specimens)	'Dog-Bone' Test Age ¹ (days)	V_f (%)	<i>f_c</i> , ² (MPa)	f_t ' (MPa)	$\frac{\mathcal{E}_t}{(x \ 10^{-3} \ \text{mm/mm})}$	E _{ct} (MPa)
TH3 (2)	366	0.50	48.0	4.57	0.158	32,800
TH4 (3)	360	1.00	48.6	4.09	0.167	29,400
TH5 (2)	374	1.50	50.7	4.05	0.181	24,500
TH8 (3)	346	1.00	45.1	3.81	0.151	29,500

Table 4.3 – SFRC Direct Tensile Strengths

¹ performed after completion of slab impact testing

² results from cylinder compression testing (see Section 4.1.1)

In all cases, the ultimate tensile stresses developed by the 'dog-bone' specimens occurred immediately prior to the development of the initial crack formation. Specimens containing fibre volume fractions of 1.00 % and 1.50 % did exhibit hardening-type characteristics such as formation of multiple cracks and the ability to sustain appreciable post-peak tensile stresses, yet post-peak loads were less than those measured in the pre-cracked condition. The occurrence of multiple cracking was most likely attributed to specimen re-alignment occurring after initial cracking. In many cases, the initial crack did not occur uniformly through the full cross sections of the specimens, resulting in varied crack widths on the faces of the specimens. Because pinned end conditions were used to restrain the specimens, the 'dog-bones' were free to rotate and re-

align to accommodate tensile loads carried across the non-uniform cracks. Even small degrees of end rotation will result in the development of significant bending stresses, which could have led to the development of additional cracks at axial load levels less than peak.

From the results presented in Table 4.3, no direct correlation between fibre volume fraction and tensile strength is clearly established. Because in all cases the tensile capacities were governed by the initial cracking load of the 'dog-bones,' the high degree of variance which is typically associated with that of the tensile strength of plain concretes should be expected. Additionally, the limited number of tensile specimens which was used to develop Table 4.3 was not sufficient to develop discernible trends regarding the tensile behaviour of concrete. However, when the results are compared with those obtained from the flexural tensile testing of the SFRCs (Table 4.2), it can be seen that they do somewhat correlate. The 'estimated' direct tensile strengths determined from the flexural tests, taken as $0.55 \cdot f_r$, are within the same range of values measured from the direct tension tests. Additionally, results from both test methods identified the TH8 companion specimens as low value outliers.

In Figure 4.7 the measured pre-peak stress versus strain behaviour from selected 'dog-bone' tests has been presented. The presented results span the range of fibre volume fractions used in this experimental program. Test results from all of the 'dog-bone' tests have been plotted and summarized in Appendix B. From the figure it can be seen that the fibre volume fraction had little effect on the cracking tensile strain and the tensile stiffness of the SFRCs.



Figure 4.7 – Pre-Peak Stress versus Strain Behaviour for Concrete 'Dog-Bones'

Tensile strains were calculated from the axial displacement measurements using an effective gauge length concept to account for non-uniform specimen geometry (Deluce, 2011). As a result, an effective gauge length of 285 mm was used for LVDTs mounted on the North and South faces of the 'dog-bones.' The gauge lengths for the LVDTs mounted on East and West faces of the specimens were not modified and were taken as 150 mm.

The measured tensile stress versus strain behaviours were used to estimate the initial tensile secant stiffness, E_{ct} , of the SFRCs from the following equation (Deluce, 2011):

$$E_{ct} = \frac{f_{t2} - f_{t1}}{\varepsilon_{t2} - \varepsilon_{t1}}$$
(4-2)

where,

 f_{t2} = the tensile stress corresponding to 60 % of the ultimate tensile strength, MPa;

 f_{tl} = the tensile stress corresponding to a tensile strain of 0.010 x 10⁻³ mm/mm, MPa;

 ε_{t2} = the tensile strain produced by f_{t2} , mm/mm;

 $\varepsilon_{tl} = 0.010 \text{ x } 10^{-3} \text{ mm/mm}.$

The post-peak behaviours from the direct tension tests have been presented in the form of tensile stress versus crack width opening relationships. The stress versus crack width relationship is more meaningful in the post-peak regime as opposed to an average strain concept since deformations are primarily limited to the opening of a single dominant crack. The crack width opening, w_{cr} , has been calculated using the following equation:

$$w_{cr} = \Delta_t - \Delta_{el} - \Delta_p \tag{4-3}$$

where,

 Δ_t = the measured axial deformation, mm;

 Δ_{el} = the calculated elastic deformation, mm;

 Δ_p = the calculated plastic deformation developed prior to obtaining the peak tensile stress, mm.

In the development of Equation 4-3, it has been assumed that the nonlinear pre-peak stress versus strain behaviour exhibited in Figure 4.7 results in the development of plastic tensile deformations

outside of the crack location. However, the elastic deformations can be approximated using Equation 4-4:

$$\Delta_{el} = \frac{P \cdot L_g}{A_c \cdot E_{ct}} \tag{4-4}$$

where,

P = the applied tensile force, N; L_g = the effective LVDT gauge length, mm; A_c = the cross-sectional area of the 'dog-bone,' mm²; E_{ct} = the tensile secant modulus calculated from Equation 4-2, MPa.

The plastic deformation is calculated as the difference between the measured deformation and the calculated elastic deformation corresponding to the peak load applied:

$$\Delta_p = \Delta_{peak} - \Delta_{el,peak} \tag{4-5}$$

where,

 Δ_{peak} = the measured axial deformation at peak load, mm;

 $\Delta_{el,peak}$ = the calculated elastic deformation from Equation 4-4 corresponding to peak load, mm.

From the post-peak behaviours presented in Figure 4.8, it can be seen that the fibre volume fraction had a significant impact on the concrete's post-cracking behaviour. As the fibre volume fraction increased, the concrete's ability to sustain tensile stresses increased. Additionally, it is clearly shown in Figure 4.8 that the increased fibre volumes resulted in reduction of the initial crack width opening. With 0.50 % fibre volume, the initial crack width opening was calculated as approximately 0.30 mm. As the fibre volume fraction increased to 1.00 % and 1.50 %, the initial crack width openings decreased to values of 0.15 mm and 0.08 mm, respectively. The resulting crack patterns from 'dog-bones' containing each fibre volume fraction investigated in this study have been presented in Figure 4.9.



Figure 4.8 - 'Dog-Bone' Post-Peak Stress versus Crack Width Opening Behaviour



Figure 4.9 - 'Dog-Bone' Crack Patterns

Upon completion of the 'dog-bone' tests, the fibre orientation factor within the dominant crack region was calculated for each specimen. The 'dog-bones' were separated into two pieces using the testing frame and loading method used to perform the uniaxial tension tests. Once separated, the number of visible fibres on each exposed crack surface was determined (see Figure 4.10). The equation presented by Soroushian and Lee (1990), was used to estimate the fibre orientation factor, α_f :

$$\alpha_f = \frac{N}{A_c} \cdot \frac{A_f}{V_f} \tag{4-6}$$

where,

N= the total number of fibres bridging the crack (summation of fibres on each crack face);

 A_c = the cross-sectional area of the specimen, mm²;

 A_f = the cross-sectional area of a single steel fibre, mm²;

 V_f = the fibre volume fraction.



Figure 4.10 – Exposed Fibres Bridging Dominant Crack

The estimated values for the fibre orientation factors of the 'dog-bone' specimens are presented in Table 4.4. It can be seen that as the fibre volume fraction increased from 0.50 % to 1.50 %, the fibre orientation factor decreased. This trend has been presented in the literature (Deluce, 2011; Soroushian and Lee, 1990), and can be attributed to the ability of the fibres to reorient themselves under applied vibrations used during material casting. Generally, higher fibre volume fractions result in additional resistance to fibre reorientation. Additionally, because the 'dogbone' specimens constructed in this study were cast horizontally, fibres tended to reorient in the plane of the specimen, consistent with the axially loaded direction.

Parent Slab (# specimens)	V_f (%)	Ν	$lpha_{f}$	C.V. (%)	
TH3 (2)	0.50	190	0.616	14.1	
TH4 (3)	1.00	365	0.591	6.8	
TH5 (2)	1.50	465	0.502	0.5	
TH8 (3)	1.00	332	0.538	2.8	

Table 4.4 – Mean Dog-Bone Fibre Orientation Factors

4.1.4 In-Situ SFRC Characteristics

Upon completion of the slab impact testing, concrete cores were sampled from the R/FRC slabs and were used to estimate the in-situ steel fibre orientation factors. The cores, which were sampled from multiple locations within the R/FRC slabs, were saw-cut into six equal sections (see Figure 4.11). Fibres exposed from the cut surfaces were counted and used to obtain estimates of the in-plane and the out-of-plane fibre orientation factors through the thicknesses of the R/FRC slabs.



Figure 4.11 - Slab Core Samples used to Study Fibre Composition

In total, sixteen core samples were collected from three of the R/FRC slabs (TH4, TH5, and TH8) and were used to create eighty cross section samples. The computed fibre orientation factors from all eighty sections are presented in Figure 4.12.



Figure 4.12 – Slab Core Sample Fibre Orientation Factors

The average value of the in-plane fibre orientation factor was estimated to be approximately 0.51 and the average out-of-plane value was estimated to be approximately 0.28. It can be seen from Figure 4.12 that although there were high degrees of scatter in the computed fibre orientation factors, the steel fibres were found to be evenly dispersed through the thicknesses of the slabs. The preferential fibre orientation in the planar direction is in agreement with theoretical relationships proposed by others (Stroeven, 1977), and has been shown to occur as a result of boundary effects and mechanical vibration used during the casting process (Gettu et al., 2005).

4.2 Drop-Test Observations

This section presents key observations made over the course of the slab testing program. Test observations were primarily in the form of developed cracking patterns, residual crack widths, mass penetration, and concrete scabbing. Note that each impact event has been identified using the slab specimen's name, followed by the impact number (i.e., TH2-2 denotes the second impact event for slab TH2). All of the slabs were subjected to the impact loading protocol presented previously in Section 3.5. Additionally, the longitudinal reinforcement ratios, ρ_l , and the fibre volume fractions, V_f , are provided for each specimen in the section headings. Selected test photos and crack map illustrations are presented within this section. Cracks within the corner support regions of the slabs which were not visible during testing are not included in the figures. Crack map illustrations pertaining to each impact event are provided in Appendix C.

4.2.1 Slab TH1 ($\rho_l = 0.420$ %; $V_f = 0$)

Slab TH1 was the first slab to be tested in the experimental program, and was constructed primarily to serve as a pilot specimen. A total of three impacts were performed on the slab, with drop-weights ranging from 120 kg to 180 kg. Based upon the test results obtained from slab TH1, the loading protocol was revised, and several modifications were made to the instrumentation provided on the slabs and drop-weight. The residual cracking patterns and widths were marked after all three impact events performed.

TH1-1 (Test Date: February 09, 2011; Drop-Weight: 120 kg)

A fanned cracking pattern on the top and bottom surfaces of the slab was observed (see Figure 4.13). The maximum residual crack width was 0.35 mm and was located on the bottom surface of the slab, in a circumferential crack immediately outside of the impact region. The majority of

the cracks were hairline, with widths of approximately 0.05 mm on the top surface, and widths ranging from 0.05 mm to 0.10 mm on the bottom surface. No visible mass penetration or concrete scabbing was observed.

Shrink-tubing connecting the potentiometers to the slab within the impact region ruptured under the impulsive loading . As a result, displacement data were not captured for potentiometers P1, P2, P6, P7, P13, and P14. Multiple layers of shrink-tubing were used to form the extension rod connection for the following test (TH1-2).



Figure 4.13 – TH1-1 Residual Crack Pattern

TH1-2 (Test Date: February 16, 2011; Drop-Weight: 150 kg)

New radial and circumferential cracks on both surfaces of the slab developed, with radial cracks occurring predominantly along the path of the reinforcing bars. A pronounced concrete punching region was visible, with significant scabbing occurring at the punching interface. The maximum residual crack width was found to be 1.0 mm, occurring within the circumferential crack bounding the punched region. Outside of the punching region, crack widths on the bottom surface remained relatively small, ranging from 0.05 mm to 0.20 mm. Residual widths on the top surface ranged from 0.05 mm to 0.10 mm throughout. The maximum mass penetration was

measured to be in the order of 1.0 mm to 1.5 mm, occurring locally at the corner of the square impact region. Photos of the damaged specimen are presented in Figure 4.14.

Even with several layers of shrink-tubing used to form the connection between the extension rods and the potentiometers, the shrink-tubing used to mount the potentiometers within the impact region ruptured. Displacement data were not captured for potentiometer P1. Additionally, signs of fatigue were observed in the shrink-tubing connecting P2 to its extension rod.



Figure 4.14 – TH1-2 Slab Damage

As a result of the frequent rupturing of the shrink-tubing used to form the connection between the potentiometers and the slab, the tubing was replaced with threaded brass couplers. The couplers were used at both ends of the extension rods, and were used for all potentiometers measuring transverse displacements. Accelerometers A6 and A11 were disconnected and removed from the bottom surface of the specimen to prevent damage. Additionally, in an effort to prevent signal saturation, a neoprene pad was installed between the mounting block for accelerometer A13 and the drop-weight.

TH1-3 (Test Date: February 25, 2011; Drop-Weight: 180 kg)

Few new cracks developed. Cracking behaviour was primarily limited to increased crack widths, and the extension of existing cracks. The maximum residual crack width was 2.5 mm occurring on the bottom surface of the slab. The majority of the crack widths ranged from 0.10 mm to 0.30 mm on the bottom surface, and remained in the range of 0.05 mm to 0.10 mm on the top

surface. Large pieces of concrete scabbed from the bottom surface of the slab exposing the bottom reinforcing bars surrounding the impact region. The maximum penetration under the third impact increased to approximately 3.5 mm. The failed slab is presented in Figure 4.15.



(a) top surface

(b) bottom surface



4.2.2 Slab TH2 ($\rho_l = 0.420$ %; $V_f = 0$)

Slab TH2 was nominally identical to slab TH1 however it did not contain out-of-plane shear reinforcement surrounding the impact region. Based upon the results obtained from the testing of TH1, the initial drop-weight was increased from 120 kg to 150 kg. TH2 was subjected to a total of three impacts, ranging from 150 kg to 210 kg. The residual cracking patterns, and widths, were marked after each impact event.

TH2-1 (Test Date: March 29, 2011; Drop-Weight: 150 kg)

A fanned cracking pattern was observed. Cracks primarily extended along the path of the reinforcing bars, with more densely spaced cracks surrounding the mass impact region (see Figure 4.16). The maximum crack width was 0.50 mm, and was located on the bottom of the slab within a circumferential crack surrounding the impact location. The majority of the residual widths were in the range of 0.05 mm to 0.20 mm on the bottom surface of the slab, and 0.05 mm

to 0.10 mm on the top surface of the slab. No mass penetration was apparent. Some localized concrete spalling occurred on the top surface of the slab within the impact location.

Potentiometers P1, P7, and P13 were damaged during the test. Data obtained from these sensors were discarded.



Figure 4.16 – TH2-1 Cracking Pattern

TH2-2 (Test Date: April 08, 2011; Drop-Weight: 180 kg)

New circumferential cracks developed on both faces of the specimen, and the residual widths of radial cracks increased. Extensive scabbing occurred circumferentially surrounding the impact region exposing the bars forming the bottom mat of reinforcement (see Figure 4.17). The maximum crack width, which occurred on the bottom surface of the slab, remained 0.50 mm. The maximum mass penetration was approximately 3.0 to 3.5 mm, and occurred locally within the mass impact region.

TH2-3 (Test Date: April 15, 2011; Drop-Weight: 210 kg)

Few new cracks developed on the slab. Damage was primarily limited to additional scabbing and increased mass penetration. Due to the extensive level of damage within, and surrounding, the impact region, light was visible through large diagonal cracks extending from the edge of the

applied impact on the top surface of the slab to the scabbed regions on the bottom surface. The maximum crack width on the bottom surface was 0.60 mm and was located at the slab's midpoint. Most cracks on the top surface were in the order of 0.10 mm to 0.20 mm. The maximum impact penetration was measured to be approximately 20 mm, with extensive penetration occurring over the full perimeter of the impact region. Longitudinal reinforcement forming the top layer of reinforcement was exposed as a result of penetration damage. The slab damage and cracking pattern is presented in Figure 4.18.



Figure 4.17 – TH2-2 Slab Damage



(a) top surface

(b) bottom surface



4.2.3 Slab TH3 ($\rho_l = 0.420$ %; $V_f = 0.50$ %)

Slab TH3 was nominally identical to slab TH2, with the exception that SFRC with a fibre volume fraction of 0.50 % was used in place of nonfibrous conventional concrete. Five impact events were performed on slab TH3, with impact mass levels ranging from 150 kg to 240 kg. The residual cracking patterns, and widths, were marked after each impact event, with the exception of TH3-4.

TH3-1 (Test Date: May 03, 2011; Drop-Weight: 150 kg)

A dense fan pattern of hairline cracks was observed. Cracks oriented along the path of reinforcing bars were comprised of multiple, tightly spaced, hairline cracks (see Figure 4.19). Most cracks were in the range of 0.05 mm to 0.15 mm on the bottom surface, and 0.05 mm to 0.10 mm on the top surface. The maximum crack width was measured to be 0.20 mm, which occurred on the bottom surface of slab across a crack in the radial direction. No visible mass penetration and no concrete scabbing were observed.



Figure 4.19 – TH3-1 Residual Crack Pattern

Due to incorrect software settings applied for the MGCplus data acquisition system, all data captured using this system (potentiometer and strain gauge data) were automatically filtered with a low frequency filtering algorithm for impact event TH3-1. Residual displacements and plastic

strains from the impact event were considered valid, all other data recorded using the MGCplus system were discarded.

TH3-2 (Test Date: May 06, 2011; Drop-Weight: 180 kg)

New cracks developed on both surfaces of the slab. On the bottom surface, multiple tightly spaced radial cracks extended from the slab's midpoint to the corners. Crack widths on the bottom surface ranged from 0.05 mm to 0.20 mm. New cracks, primarily in the form of circumferential cracks developed on the top surface of the slab. The cracks were comprised of multiple tightly spaced hairline cracks, and were located adjacent to the supports. No visible concrete scabbing was observed. The maximum mass penetration was measured to be approximately 2.0 mm, occurring locally under the impacting mass.

TH3-3 (Test Date: May 10, 2011; Drop-Weight: 210 kg)

The formation of new, relatively small, cracks was observed throughout. Residual cracks on the top surface of the slab remained in the range of 0.05 to 0.15 mm; crack widths as large as 0.50 mm were observed adjacent to the impact location. On the bottom surface of the slab, the maximum crack width was 0.80 mm, and was located in a circumferential crack outside of the impact region. No concrete scabbing occurred; maximum penetration increased to approximately 4.5 mm. The cracking behaviour and mass penetration on the slab's top surface is presented in Figure 4.20.

Potentiometers P7 and P14 were damaged during the impact event. Displacement data from these sensors were discarded.

TH3-4 (Test Date: June 03, 2011; Drop-Weight: 240 kg)

Few new cracks were observed. Limited scabbing was observed on the bottom surface of the slab, occurring within a single circumferential crack surrounding the impact location. The maximum mass penetration increased to approximately 8.0 mm and was found to occur over the full perimeter of the impacting mass.

Strain gauge S2 was damaged during event TH3-4. The data from this sensor were discarded.



(a) top surface (b) mass penetration Figure 4.20 – TH3-3 Damage

TH3-5 (Test Date: June 03, 2011; Drop-Weight: 240 kg)

The development of additional scabbing within a single circumferential crack on the bottom surface of the slab was observed. The maximum crack width on the bottom surface was 2.0 mm and was located at the slab's midpoint. Crack widths on the top surface remained relatively small, with the majority ranging from 0.05 to 0.15 mm. The mass penetration occurred along the full perimeter of the impact location and was measured to be approximately 25 mm. The slab damage and cracking pattern is presented in Figure 4.21.



(a) top surface (b) bottom surface Figure 4.21 – TH3 Final Cracking Pattern

4.2.4 Slab TH4 ($\rho_l = 0.420$ %; $V_f = 1.00$ %)

A fibre volume fraction of 1.00 % was used in the SFRC comprising slab TH4. A total of seven impact tests were performed on the slab, with drop-weights ranging from 150 kg to 270 kg. The residual cracking patterns, and widths, were marked for impact events TH4-1 through TH4-3, TH4-5, and TH4-7.

TH4-1 (Test Date: June 14, 2011; Drop-Weight: 150 kg)

A fanned cracking pattern consisting of predominantly hairline cracks was observed. The majority of the crack widths on the bottom surface ranged from 0.05 to 0.10 mm. The widest crack, which occurred radially on the bottom surface, was measured to be 0.20 mm. On the top surface, nearly all of the crack widths were measured to be 0.05 mm or less. In a few occurrences, residual widths of 0.10 mm were measured on the top surface of the slab. After the first impact event, no mass penetration or material scabbing was observed.

TH4-2 (Test Date: June 16, 2011; Drop-Weight: 180 kg)

The second impact on slab TH4 resulted in the development of new radial and circumferential cracks on both surfaces of the slab. The majority of the cracks were small in width. On the top surface of the slab, crack widths of 0.20 mm occurred locally at the location of the impacting mass. The maximum crack width on the bottom surface of the slab was 0.35 mm and occurred in a radial crack, outside of the impact region. Localized mass penetration of approximately 1.0 mm was observed. The residual crack widths are presented in Figure 4.22.

Potentiometers P2, P6, and P14 were damaged during the test, and the data were discarded.

TH4-3 (Test Date: June 21, 2011; Drop-Weight: 210 kg)

The development of a main circumferential crack occurring immediately outside of the impact region on the bottom surface of the slab was observed. The width of the circumferential crack was as large as 0.90 mm in some locations; however, the majority of the crack widths on the bottom surface remained in the range of 0.10 to 0.20 mm. No increases in the crack widths on the top surface of the slab were observed. The maximum mass penetration was approximately 2.0 to 2.5 mm. No concrete scabbing was observed. The residual crack pattern is presented in Figure 4.23.



(a) top surface

(b) bottom surface





(a) top surface

(b) bottom surface

Figure 4.23 – TH4-3 Residual Crack Pattern

TH4-4 (Test Date: June 23, 2011; Drop-Weight: 240 kg)

Localized scabbing was observed at the location of the circumferential crack developed from impact TH4-3. The maximum mass penetration increased to 6.0 mm, and occurred relatively uniformly over the perimeter of the impact region.

TH4-5 (Test Date: June 23, 2011; Drop-Weight: 240 kg)

The main circumferential crack on the bottom surface increased to a maximum residual width of 1.20 mm. Radial crack widths along the path of the reinforcing bars were comprised of multiple tightly spaced cracks, with maximum widths in the range of 0.25 to 0.50 mm (see Figure 4.24b). Cracks on the top surface had widths ranging from 0.05 to 0.15 mm, with the maximum widths occurring outside of the support locations. The measured mass penetration remained approximately 6.0 mm, and was uniform over the impacting area (see Figure 4.24a).



(a) mass penetration

(b) radial cracks

Figure 4.24 – TH4-5 Damage

TH4-6 (Test Date: June 28, 2011; Drop-Weight: 270 kg)

Additional localized scabbing from the location of the main circumferential crack was observed. The mass penetration increased to approximately 11.0 mm.

Potentiometer P7 was removed due to its mounting dowel coming unanchored. The data from P7 were discarded, and the sensor was not reconnected for the following impact tests on slab TH4.

TH4-7 (Test Date: June 28, 2011; Drop-Weight: 270 kg)

Fibres bridging the areas of localized scabbing were observed. Scabbing was predominantly limited to the main circumferential crack location, which was also the area of the largest residual crack widths with a maximum measured value of 4.0 mm. Large cracks distributed throughout

the bottom surface of the slab in the range of 0.35 to 0.65 mm were also visible in radial cracks. The majority of the cracks on the top surface of the slab were in the range of 0.10 to 0.20 mm. A maximum crack width of 0.60 mm was measured outside of the punched area on the top surface of the slab. Mass penetration was measured to be approximately 17 to 18 mm, and occurred throughout the impact area. The slab damage and cracking patterns are presented in Figure 4.25.



(a) top surface

(b) bottom surface

Figure 4.25 – TH4 Final Cracking Pattern

4.2.5 Slab TH5 ($\rho_l = 0.420$ %; $V_f = 1.50$ %)

Slab TH5 was comprised of SFRC with a fibre volume fraction of 1.50 %, the largest fibre volume fraction investigated in this study. The slab contained identical reinforcement ratios to those of slabs TH1 through TH4. In total, slab TH5 was subjected to ten impact tests with drop-weights ranging from 150 kg to 300 kg. It was anticipated that slab TH5 would be capable of enduring several impacts due to the increased fibre content. As such, the residual cracking patterns and widths were marked after events TH5-1, TH5-3, TH5-5, TH5-7, and TH5-10.

TH5-1 (Test Date: July 07, 2011; Drop-Weight: 150 kg)

An initial fanned cracking pattern consisting of small discontinuous cracks was observed on the bottom surface (see Figure 4.26). Cracks were primarily focused along the paths of the

reinforcing bars and had widths ranging from 0.05 to 0.10 mm. The maximum crack width on the bottom surface was 0.15 mm occurring in a radial crack. Limited cracking with large uncracked areas of concrete was observed on the top surface of the slab. All cracks on the top surface had residual widths of 0.05 mm or less. No visible mass penetration or concrete scabbing was observed.



Figure 4.26 – TH5-1 Residual Crack Pattern

TH5-2 (Test Date: July 12, 2011; Drop-Weight: 180 kg)

New radial and circumferential crack development on both faces of the slab was observed. The cracks appeared to be of uniform widths on each surface of the slab. Mass penetration occurred locally within the impact region, and was measured to be approximately 1.0 mm.

Two potentiometers were damaged during the test: P1 and P13. The potentiometers were replaced and the acquired data were discarded.

TH5-3 (Test Date: July 12, 2011; Drop-Weight: 210 kg)

The development of a circumferential crack with a maximum width of 0.45 mm was observed on the bottom surface. Outside of the main circumferential crack, the majority of the cracks on the bottom surface of the slab had widths ranging from 0.05 to 0.20 mm. Many new small cracks
developed on the top surface of the slab with widths ranging from 0.05 to 0.10 mm. The cracks on both surfaces were primarily focused along the paths of the reinforcing bars, and were closely spaced together. The maximum mass penetration increased to approximately 2.0 to 2.5 mm; no concrete scabbing was observed. The measured crack widths on the top and bottom surfaces of the slab are presented in Figure 4.27.

Potentiometer P13 was damaged. The instrument was replaced and the data collected from P13 were discarded.



(a) top surface

(b) bottom surface

Figure 4.27 – TH5-3 Damage

TH5-4 (Test Date: July 15, 2011; Drop-Weight: 240 kg)

The fourth impact test resulted in little additional damage. Cracking behaviour was primarily limited to crack extensions, no evidence of concrete scabbing was observed. The mass penetration was in the range of 2.5 to 3.0 mm.

TH5-5 (Test Date: July 15, 2011; Drop-Weight: 240 kg)

Residual widths of the radial cracks on the bottom surface of the slab increased. The width of the previously developed circumferential crack did not increase significantly. The maximum crack width was 0.90 mm and occurred in a radial crack outside of the punching region. The majority of the cracks on the bottom surface were in the range of 0.05 to 0.20 mm. The cracks on the top surface had widths ranging from 0.05 to 0.10 mm. No concrete scabbing was observed and the

maximum mass penetration was approximately 3.5 to 4.0 mm, occurring locally in the southwest corner of the impact region. The residual crack pattern is presented in Figure 4.28.



(a) top surface (b) bottom surface Figure 4.28 – TH5-5 Residual Crack Pattern

TH5-6 (Test Date: July 18, 2011; Drop-Weight: 270 kg)

On the bottom surface, new small cracks developed adjacent to existing wide cracks. Crack widths appeared to grow uniformly amongst circumferential and radial cracks. The mass penetration increased to 4.5 mm.

TH5-7 (Test Date: July 19, 2011; Drop-Weight: 270 kg)

Crack widths increased throughout the bottom surface of the slab (see Figure 4.29). The maximum crack width was 1.60 mm, and occurred within a radial crack. The cracks on the top surface of the slab remained relatively small, ranging from 0.05 to 0.15 mm, with the majority of the cracks being less than or equal to 0.10 mm. Localized mass penetration in the order of 4.5 to 5.0 mm was observed. No concrete scabbing was observed.

TH5-8 (Test Date: July 21, 2011; Drop-Weight: 300 kg)

On the bottom surface of the slab, new small radial cracks developed adjacent to existing cracks. Existing circumferential cracks showed little evidence of growth or extension.



Figure 4.29 – TH5-7 Residual Crack Widths (bottom surface)

TH5-9 (Test Date: July 21, 2011; Drop-Weight: 300 kg)

The radial cracks on the bottom surface and the sides of the slab were visibly wider. Steel fibres were exposed, bridging large cracks throughout the bottom surface of the slab. The maximum mass penetration increased to approximately 6.0 to 6.5 mm.

TH5-10 (Test Date: July 21, 2011; Drop-Weight: 300 kg)

After the tenth impact, crack widths throughout the bottom surface had increased significantly. Multiple cracks had widths in excess of 2.0 mm, the maximum crack width was 3.5 mm. Residual crack widths on the top of the slab remained small, ranging from 0.05 to 0.15 mm, with cracks greater than 0.10 mm occurring in few localized locations. No dominant circumferential crack indicating a punching type failure was evident, and no concrete scabbing was observed. The maximum penetration remained in the order of 6.0 to 6.5 mm, and occurred locally in the southwest corner of the impact region. The slab displayed excessive permanent deformations. The final cracking pattern for TH5 is presented in Figure 4.30.

4.2.6 Slab TH6 ($\rho_l = 0.273$ %; $V_f = 0$)

Slab TH6 was constructed using conventional concrete. It was the most lightly reinforced slab, with a longitudinal reinforcement ratio of 0.273 %. The slab was subjected to two impacts: 150 kg and 180 kg. The residual cracking patterns and widths were marked after both impact events.

No data were acquired for strain gauges S8 and S10 due to damage obtained during casting.



(a) top surface

(b) bottom surface

Figure 4.30 – TH5 Final Cracking Pattern

TH6-1 (Test Date: July 27, 2011; Drop-Weight: 150 kg)

The development of a few large cracks along the path of the reinforcing bars on the bottom surface of the slab was observed (see Figure 4.31). On average, the cracks ranged from 0.15 to 0.35 mm on the bottom surface. Cracks on the top surface of the slab were small, ranging from 0.05 to 0.15 mm. Localized penetration within the impact region occurred, measuring approximately 3.0 to 3.5 mm in depth.

Potentiometer P1 was damaged. The data were discarded and the sensor was replaced.

TH6-2 (Test Date: July 28, 2011; Drop-Weight: 180 kg)

Few new cracks developed. Cracking behaviour was primarily limited to the extension of existing cracks and increased widths. As such, the cracks remained concentrated at the locations of the reinforcing bars on both surfaces of the slabs. Residual widths ranged from 0.05 to 0.20 mm on the top surface of the slab, and ranged from 0.10 to 0.50 mm on the bottom surface. The maximum crack width was 0.85 mm, and occurred within a radial crack, outside of the punching region. Extensive scabbing occurred circumferentially around the region of impact. Bars from the bottom mat of reinforcement were exposed, and light was visible through the

depth of the slab. The mass penetration increased to approximately 18 to 20 mm, and was uniform over the impact region. The failed slab is presented in Figure 4.32.

The data from potentiometers P2, P8, and P19 were discarded due to damage.



(a) top surface (b) bottom surface Figure 4.31 – TH6-1 Residual Crack Pattern



(a) top surface

(b) bottom surface Figure 4.32 – TH6 Failed Slab

4.2.7 Slab TH7 ($\rho_l = 0.592$ %; $V_f = 0$)

Slab TH7 was constructed with conventional, nonfibrous concrete. It contained the largest longitudinal reinforcement ratio investigated in the experimental program: 0.592 % pertaining to each layer of reinforcement. A total of three impact tests were performed on TH7, with drop-weights ranging from 150 kg to 210 kg. Residual crack patterns and widths were marked after each impact event performed.

During the testing of TH7, no data were acquired for strain gauge S4 due to sensor damage obtained during the casting process.

TH7-1 (Test Date: August 09, 2011; Drop-Weight: 150 kg)

After the initial impact, cracks developed along the paths of the reinforcing bars on both surfaces of the slab. The most distinct crack occurred on the bottom surface, circumferentially around the impact region (see Figure 4.33). The main circumferential crack had a maximum crack width of 0.60 mm. Most other cracks on the bottom surface were in the range of 0.05 to 0.15 mm wide. Cracks on the top surface ranged from 0.05 to 0.15 mm, and did not develop to the same extent as the bottom surface cracks. No signs of mass penetration or concrete scabbing were observed.



Figure 4.33 – TH7-1 Residual Crack Pattern

TH7-2 (Test Date: August 10, 2011; Drop-Weight: 180 kg)

Increased crack widths in the circumferential cracks surrounding the impact region on the bottom of the slab were observed. The residual width of the main punching crack increased to a maximum value of 1.8 mm (see Figure 4.34). Cracks outside of the punching region remained relatively small, ranging from 0.05 to 0.20 mm. Damage to the top surface was in the form of extension of the existing cracks. The crack widths on the top surface remained relatively small, ranging from 0.05 to 0.15 mm. Localized within the main punching crack, limited scabbing was observed. The maximum mass penetration was approximately 9 to 10 mm, which occurred throughout the impact region.

Data from potentiometers P1, P5, P6, and P7 were discarded due to sensor damage or anchor failure between the potentiometer's extension rod and the slab. Based on the anticipated punching behaviour of slab TH7, and damage levels encountered from impact event TH7-2, several potentiometers were removed prior to performing test TH7-3: P7, P8, P9, P14, P15, P17, P18, and P19.



Figure 4.34 – TH7-2 Damage

TH7-3 (Test Date: August 12, 2011; Drop-Weight: 210 kg)

The final impact test on TH7 resulted in extensive damage to the slab within the impact region. Concrete scabbing occurred circumferentially around the impact region, exposing the bottom mat of reinforcement. The mass penetrated the slab uniformly over the impact region to a depth of 30 mm. The maximum crack width was found to be 2.0 mm and occurred on the bottom of the slab within the impact region. Crack widths on the top of the slab remained relatively small, ranging from 0.05 to 0.15 mm. The slab's crack pattern is presented in Figure 4.35.

As noted above, potentiometers P7, P8, P9, P14, P15, P17, P18, and P19 were removed prior to performing TH7-3 to minimize sensor damage. As such no data were acquired from these sensors. Additionally, sensor P6 became detached during the impact event, and no viable data were acquired from the sensor.



(a) top surface (b) bottom surface Figure 4.35 – TH7 Final Cracking Pattern

4.2.8 Slab TH8 ($\rho_l = 0.592$ %; $V_f = 1.00$ %)

The longitudinal reinforcement of slab TH8 was nominally identical to that of TH7 and represents the largest longitudinal reinforcement ratio investigated in this program. The slab was comprised of SFRC with a fibre volume fraction of 1.00 %. A total of eight impact tests were performed on TH8, with drop-weights ranging from 150 kg to 300 kg. As in the case of slab TH5, it was anticipated that slab TH8 would require several impacts prior to sustaining significant levels of damage. As such, the residual cracking patterns, and widths, were only marked after impact events TH8-1, TH8-3, TH8-5, TH8-7, and TH8-8.

TH8-1 (Test Date: August 18, 2011; Drop-Weight: 150 kg)

A fanned cracking pattern consisting of small hairline cracks was observed (see Figure 4.36). The majority of the cracks on the bottom surface ranged from 0.05 to 0.10 mm. The widest crack, which occurred radially on the bottom surface, was measured to be 0.20 mm. On the top surface, nearly all of the cracks were measured to be 0.05 mm or less. In a few occurrences, residual widths of 0.10 mm were measured on the top surface of the slab. After the first impact event, no mass penetration or material scabbing was observed.

Data from potentiometer P1 were discarded due to sensor damage. The sensor was replaced prior to performing the next impact event.



Figure 4.36 – TH8-1 Residual Crack Pattern

TH8-2 (Test Date: August 23, 2011; Drop-Weight: 180 kg)

New cracks primarily in the form of crack extensions were observed. Cracks appeared to be of similar widths to those measured after impact event TH8-1. No concrete scabbing was observed on the bottom surface of the slab. Localized penetration ranging from 0.50 to 1.0 mm was measured in the corners of square impacting region.

Potentiometers P1 and P6 were damaged during the impact. The data from these sensors were discarded, and the sensors were replaced prior to the performance of the next impact event.

TH8-3 (Test Date: August 23, 2011; Drop-Weight: 210 kg)

New cracks primarily in the form of circumferential cracks developed between already existing radial cracks on both surfaces of the slab (see Figure 4.37). The residual widths of pre-existing cracks increased slightly; however, maximum crack widths remained 0.15 mm on the top surface, and 0.20 mm on the bottom surface. No concrete scabbing was observed on the bottom surface of the slab. The maximum mass penetration increased to a depth of 5.0 mm which occurred locally at the corner of the impact region. The average penetration was in the range of 1.0 to 1.5 mm.

During the impact event, numerous dowels anchoring the potentiometer extension rods were damaged. As a result, data from potentiometers P1, P4, P6, and P7 were discarded.



Figure 4.37 – TH8-3 Damage

TH8-4 (Test Date: August 26, 2011; Drop-Weight: 240 kg)

Under the fourth impact, few new cracks were observed. Some localized material loss in the form of concrete flaking from the bottom surface occurred, although no significant material scabbing was observed. The mass penetration increased to an average value of 2.0 to 2.5 mm which occurred uniformly along the impact perimeter. The slab damage is presented in Figure 4.38.



(a) top surface (b) bottom surface Figure 4.38 – TH8-4 Slab Damage

TH8-5 (Test Date: August 26, 2011; Drop-Weight: 240 kg)

The first signs of significant punching were apparent. Localized scabbing occurred circumferentially around the impact region on the bottom surface of the slab (see Figure 4.39). The crack widths on the bottom surface of the slab increased significantly, measuring as large as 1.1 mm in the circumferential crack forming the punched region, and as large as 0.70 mm in the radial cracks. Most cracks on the bottom surface remained in the range of 0.10 to 0.20 mm. The crack widths on the top surface of the slab remained small, ranging from 0.05 to 0.10 mm. The mass penetration increased to a range of 3.0 to 3.5 mm.



(a) top surface (b) bottom surface Figure 4.39 – TH8-5 Residual Crack Pattern

Data from P1, P6, and P7 were lost due to sensor damage and dislodging of the dowels.

TH8-6 (Test Date: August 30, 2011; Drop-Weight: 270 kg)

The localized scabbing surrounding the impact region increased slightly. Radial cracks appeared to increase in width. Few new cracks were apparent. The mass penetration was uniform along the impact perimeter, and ranged from 8.0 to 8.5 mm.

TH8-7 (Test Date: August 30, 2011; Drop-Weight: 270 kg)

Most new cracks were small in width and occurred alongside already existing cracks. The maximum crack width on the bottom surface of the slab was 1.1 mm and was located within a radial crack. The majority of the cracks on the bottom surface of the slab had widths ranging from 0.15 to 0.25 mm. Concrete scabbing, which had occurred circumferentially around the impact region, increased considerably. Cracks on the top surface remained small with widths ranging from 0.05 to 0.20 mm. The mass penetration increased to approximately 12 mm (see Figure 4.40).

Data from sensors P1 and P2 were lost, resulting in no transverse displacement for the midpoint of the slab. Prior to performing impact event TH8-8, sensors P1 and P2 were re-connected and a number of potentiometers located in regions of scabbing were removed to prevent further damage. The following sensors were removed prior to performing event TH8-8: P5, P6, P7, P13, P14, P18, and P19.



Figure 4.40 - TH8-7 Mass Penetration

TH8-8 (Test Date: September 01, 2011; Drop-Weight: 300 kg)

After the eighth and final impact performed on TH8, the slabs surfaces were cracked extensively (see Figure 4.41). Maximum widths on the bottom surface ranged from 1.2 to 1.4 mm and occurred at several locations within radial cracks throughout the slab. The scabbing on the bottom surface of the slab increased, but remained in the location of the circumferential crack forming the punched region. Cracks on the top surface of the slab remained small, with the majority ranging from 0.10 to 0.15 mm. The maximum mass penetration was measured as 25 mm, which occurred locally within the corner of the impact region. The majority of the penetration was in the range of 18 to 20 mm.

No potentiometers were damaged during impact event TH8-8; however, as noted above, a reduced number of potentiometers were used during the test.



(a) top surface

(b) bottom surface

Figure 4.41 – TH8 Final Cracking Pattern

4.3 Digital Data Processing

This section presents response-time histories from the various data types acquired during the slab impact testing: displacements, loads, strains, and accelerations. Because an extremely large volume of data was acquired from this testing program, only selected results from individual impact events have been presented in the following section. Additional presentations of the acquired digital data are provided in Appendix C. The unfiltered digital data can be accessed online at <u>http://www.civ.utoronto.ca/vector/</u>.

The data presented within this section have been selected to encompass the varied slab responses observed throughout the testing program. Presentation of results from individual impact events are limited, and are used primarily to assess the quality and validity of the acquired digital data.

A primary concern in the performance of any type of dynamic testing program is the ability to accurately and sufficiently measure data which are used to characterize test results: displacements, loads, etc. In the case of reinforced concrete under impact or shock, additional challenges are realized due to the extremely short event durations and high frequency behaviours which are often exhibited. In this experimental program, digital data were sampled at discrete time intervals from continuous analog signals provided by the instruments used in the study (see Section 3.4). According to the Nyquist Sampling Theorem, if the digital data are sampled at a frequency which is greater than twice the maximum exhibited response frequency (i.e., the 'Nyquist frequency'), then a continuous analog signal can be reconstructed without developing an aliased response (Stein, 2000). Because in this experimental program data are used to represent events which occur over very small time periods and are used to reassemble highly non-uniform responses, a sampling rate frequency several times larger than the Nyquist frequency is appropriate and was used to ensure that reassembly of the digital data sufficiently captured measured peak values.

The following sections present brief discussions pertaining to the data collected by the various types of instruments used in this experimental program. Similar discussions pertaining to the validity of acquired impact data were provided by Saatci (2007), and were found to be useful in assessing the limitations of provided instrumentation. It is important to note that the developed power spectrums presented in this section represent 'measured' frequency content and not necessarily that exhibited by the test slabs. However, in most cases, the relationships were sufficient to validate the sampling rates used in this program. Additional discussion regarding the use of low-pass filtering to condition acquired digital data is also provided.

4.3.1 Displacement Data

This section presents the digital displacement data obtained from the potentiometers. The discussion is focused primarily on the midpoint displacement-time histories of the slabs. Discussion regarding the deformation characteristics of the slabs is provided in Section 4.4.

The midpoint displacement response of beams and slabs under impact is often used to characterize overall specimen behaviour. Such approaches should be viewed with caution, specifically with regard to reinforced concrete, since the occurrence of non-uniform, localized behavioural characteristics arising from inelastic material properties may not be apparent from the midpoint response. However, for the purpose of presenting a brief overview of the captured responses and assessing the validity of the acquired displacement data, as is done in this section, the midpoint displacement response will be used explicitly.

The displacement data have been analyzed in terms of their frequency content using the FFT (Fast Fourier Transform) method to develop power spectrums from the acquired displacement responses. FFT is a commonly used tool in digital signal analysis and, in this case, the development of the power spectrum will be used to determine dominant frequencies comprising the digital response. The FFT was performed using the Catman[®] software program developed by HBM, the same software program serving as the interface for the data acquisition systems. No discussion pertaining to the methodology or algorithms used to carry-out the FFT is provided in this thesis. However, information regarding the application and performance of the method is readily available elsewhere (Rao et al., 2009; Blahut, 2010).

The results from four impact events representing different damage levels have been presented in this section. Impact events TH5-1 (see Figure 4.42) and TH7-1 (see Figure 4.43) represent low mass impacts on high stiffness, undamaged slabs. Event TH6-2 (see Figure 4.44) represents an impact event performed on a damaged slab, resulting in extensive additional damage. Lastly, impact TH5-10 (see Figure 4.45) presents a high mass impact event on an extensively damaged slab. Secondary responses due to mass rebounding captured in the digital data set have not been included in the following figures, and were not used to develop the power spectrums. Additionally, the midpoint-displacement responses do not consider the accumulation of residual midpoint displacements from prior impacts.









Figure 4.45 – Midpoint Displacements, Event TH5-10

From the results presented in Figure 4.42 through Figure 4.45, it can be seen that the predominant frequency pertaining to the midpoint transverse displacement ranged from approximately 28 to 38 Hz, corresponding to the damaged and undamaged slabs, respectively. Additionally, it is apparent from the figures that displacement components with frequencies greater than approximately 200 Hz contribute little to the overall midpoint displacement response. Lastly, from Nyquist's Sampling Theorem, the above figures suggest that a minimum data sampling frequency in the order of 400 Hz is required to ensure accurate representation of the frequency content of the measured response. As such, the sampling rate of 2,400 Hz which was used to acquire all displacement data was sufficient.

4.3.2 Load Cell Data

This section presents the digital data obtained from the load cells provided at the slab corner supports. The measured reaction loads from two of the four load cells will be presented and discussed for selected impact events. A more detailed discussion regarding the support reaction-time histories is provided in Section 4.5.

Accurate measurement of the loads involved in impact testing programs often presents significant challenges. The support reactions and applied impact loads exhibit high frequency behaviours and tend to occur over time periods less than those exhibited by the displacement responses. Because of this, the measurements from the load cells were collected at an increased sampling rate of 96 kHz, without filtering.

Similar to Section 4.3.1, selected impact events are used to represent a range of measurements encountered throughout the testing program. The presented load cell data represent the maximum and minimum peak reaction forces measured under a specific impact event. TH4-4 (see Figure 4.46) represents an impact event where a high peak reaction force was measured, and TH6-1 (see Figure 4.47) represents an impact event where a low peak reaction force was measured.

Note that the occurrence of load cell negative reaction measurements was due to the initial posttensioning forces that were applied through the tie-down assemblies comprising the support reactions (see Section 3.2). As such, the negative measurements are in fact valid, and are considered in evaluating the total reaction.





From the developed power spectrums presented in Figure 4.46 and Figure 4.47, it can be seen that the measured load cell reactions are comprised of a much broader frequency range than the midpoint displacements. In both impact events considered, predominant load cell response contributions were found to occur at frequencies of approximately 23 Hz and 152 Hz. The range of frequencies which were determined to contribute significantly to the load cell response was from zero to approximately 800 Hz. For clarity, the load cell response spectrums have been presented over a reduced frequency range of 1,200 Hz. It should be noted that no additional response contributions were identified for frequencies from 1,200 Hz to 48 kHz.

The load cell responses from impact TH4-4 are again presented in Figure 4.48 with focus on the measurement of peak load values. It can be seen that the sampling rate used for the load cells was effective in capturing high frequency oscillations and peak load cell values.



Figure 4.48 – Peak Load Cell Responses, Event TH4-4

4.3.3 Strain Data

This section presents the digital data sampled from the strain gauges used to measure the slab uplift reaction forces at the supports, and gauges installed on the reinforcing bars within the slabs. All strain gauges were sampled at a rate of 2,400 Hz.

The data obtained from two impact events will be considered: event TH6-1 is used to investigate the strain behaviour of an undamaged slab (see Figure 4.49 and Figure 4.50), and impact event

TH5-10 is used to investigate the strain characteristics of a slab that had sustained considerable damage from prior impact tests.

Data collected from gauge S1 was selected to serve as representative strain measurements sampled from the reinforcing bars. Strain gauge S1 was installed on the bottom layer of reinforcement, in the midpoint region (see Appendix A). Strain measurements collected from one of the four strain gauges used to calculate the uplift force of the South support, gauge SW1, was used to represent the gauges on the Dywidag assemblies used in the testing frame. Strain gauge SW1 was one of two gauges located on the West Dywidag bar, of the South corner support.

From Figure 4.49 and Figure 4.50, it can be seen that under impact event TH6-1 the data collected from the strain gauges on the Dywidag support bars possessed a much broader range of contributing frequencies than data sampled from the gauges installed on the reinforcing bars. The developed power spectrum for gauge S1 on the reinforcing bar suggests that predominant contributions to the response occurred up to a frequency of 28.1 Hz, and only small contributions exist for frequencies greater than approximately 400 Hz. The strain measurements obtained from TH6-1 for the Dywidag bars display much richer frequency content, with significant contributions estimated to occur up to approximately 200 Hz. Additionally, the developed power spectrum presented in Figure 4.50b suggests that minimal contributions to the strain gauge response exist throughout the full range of captured frequencies.



Figure 4.49 - Reinforcing Bar Strain Gauge; Gauge S1, Event TH6-1



Figure 4.50 - Support Bar (Dywidag) Strain Gauge; Gauge SW1, Event TH6-1

The collected strain gauge data and developed power spectrums from impact event TH5-10 are presented in Figure 4.51 and Figure 4.52. For both strain gauges it can be seen that the high frequency contributions to the measured strain responses are much less significant relative to the estimated peak response contributions for the damaged slab specimen. The reinforcing bar strain measurements are comprised primarily of response contributions with frequencies ranging from zero to 400 Hz, with a peak occurring at a frequency of 32.8 Hz. Strains measured from the Dywidag bar suggested that significant contributions to the measured response occurred over a frequency range of zero to 200 Hz.



Figure 4.51 - Reinforcing Bar Strain Gauge; Gauge S1, Event TH5-10



Figure 4.52 – Support Bar (Dywidag) Strain Gauge; Gauge SW1, Event TH5-10

From the above figures pertaining to impacts TH6-1 and TH5-10, it is believed that the sampling rate of 2,400 Hz which was used to collect the strain data was sufficient to capture overall trends and strain responses exhibited by slabs. In the case of impact event TH6-1, it is apparent that the measured strain response of the Dywidag bar (see Figure 4.50b) could have benefitted from an increased sampling rate to ensure that all high frequency response contributions were collected. Figure 4.53a presents a portion of the reinforcing bar strain response captured from impact event TH6-1. It is evident that the 2,400 Hz sampling rate was not sufficient to ensure that peak strain values were captured throughout the full response. The same conclusion can be drawn from the sampled Dywidag bar strains measured from event TH5-10 (see Figure 4.53b).



Figure 4.53 – Peak Strain Measurements

4.3.4 Acceleration Data

This section presents the digital data obtained from the accelerometers which were used to measure the accelerations of the slabs and the impacting drop-weights. The acceleration measurements presented in this section were sampled at a rate of 96 kHz, and have not been filtered. Two accelerometers, identified as A3 and A5, are used for the investigation of the acceleration response exhibited by the test slabs. Measurements from one accelerometer mounted to the drop-weight, identified as A13, are also presented in this section. Refer to Appendix A for accelerometer instrumentation plans.

Throughout the course of the testing program the measured response from one, or sometimes both, of the accelerometers mounted to the drop-weight often saturated due to resonating mass vibrations which occurred after the initial impact. In most cases, accelerometer saturation occurred after data from the initial impact was collected. However, because the initial impacts occurred over very short durations of time, insufficient samples were collected to create high resolution power spectrums of the drop-weight accelerations. Consider the data collected from A13 during impact event TH7-1 (see Figure 4.54): from the time of initial impact, approximately 2.5 milliseconds worth of meaningful acceleration data are collected prior to the response saturating, resulting in the collection of only 240 samples from the acceleration response. Such a small data set is not sufficient to create a high resolution response spectrum over the 48 kHz frequency range which could theoretically be captured from sampling at a rate of 96 kHz.



Figure 4.54 – Saturated Mass Acceleration Measurements

The acceleration data from two impact events have been investigated in this section: event TH4-1 (Figure 4.55 and Figure 4.56) representing the response from an undamaged slab, and event TH4-5 (Figure 4.57 and Figure 4.58) representing the response from a slab possessing significant damage from prior impacts.



Figure 4.56 – Drop-Weight Accelerations, Event TH4-1

From the acceleration data presented above, it can be seen that measurements acquired from the accelerometers mounted to the drop-weight and slab possessed significant contributions from high frequency components. In the case of the slab accelerations (Figure 4.55), frequencies as high as 18 kHz contribute to the measured response. Contributions to the measured mass acceleration responses occur up to approximately 12 kHz. It should be noted that the peak contributions to the measured accelerations occurred at very low frequencies.

The measured acceleration responses from event TH4-5 are presented in Figure 4.57 and Figure 4.58. It can be seen that the frequency content comprising the measured acceleration responses from event TH4-5 is of a lower frequency range than that exhibited by TH4-1. It is also interesting to note that the time differential between the initiation of the measured responses from accelerometers A3 and A5 has increased substantially from that exhibited in Figure 4.55a. In comparing the responses from the two impact events, recall that test TH4-5 was performed using an increased impacting mass and was performed on a slab that had accumulated significant prior damage. Both factors contribute to the noted differences amongst the measured slab accelerations from the two events.

The drop-weight accelerations presented in Figure 4.58 are also notably different than those presented for event TH4-1. The acceleration amplitude was substantially larger in TH4-5, and the overall response is less noisy than that presented in Figure 4.56. This is most likely due to the reduction in slab stiffness and increase in impacting mass level. In both events, the drop-weight accelerations decayed significantly immediately after initial impact.

The developed power spectrums for both the mass and the slab accelerations from test TH4-5 were found to possess significantly less high frequency content than that exhibited in TH4-1. The range of contributing frequencies decreased significantly, and the peak frequency contributions also decreased.



Figure 4.57 – Slab Accelerations, Event TH4-5



Figure 4.58 – Drop-Weight Accelerations, Event TH4-5

Based on the previous results presented within this section, it can be seen that the acceleration data were the most demanding in terms of data acquisition requirements. The measured responses were comprised of contributions spanning large frequency ranges, with significant high frequency components. The use of the 96 kHz sampling rate appears to have been successful in capturing most of the significant contributions to the acceleration responses. Investigation of the peak measurement values (see Figure 4.59) suggest that the response resolution may have improved with sampling rates greater than 96 kHz.



Figure 4.59 - Peak Acceleration Measurements, Event TH4-1

4.3.5 Data Filtering

From the selected digital data measurements presented within Sections 4.3, it can be seen that some of the collected data could benefit from some form of signal processing. In particular, the unfiltered acceleration responses were noisy, exhibited high frequency oscillations, and displayed significant variations in peak measured values. With little certainty as to which frequency contributions to the measured response are meaningful and which should be considered as artificial background noise or erroneous, interpreting and processing the data forms a significant challenge.

Many researchers investigating impact phenomena have taken the approach of acquiring data at high sampling rates and applying various filtering algorithms to remove what is believed to be erroneous response occurring within the high frequency ranges. This method of data processing was used in a previous investigation regarding the behaviour of RC slabs subject to drop-weight impacts (Chen and May, 2009). In the experimental program performed by Chen and May, all data, including slab and drop-weight accelerations, were sampled at a rate of 500 kHz. The data were filtered using a low-pass Butterworth filter with a cutoff frequency of 2 kHz. Discussion regarding the significance of the selected filtering method was not provided. Although this method of signal processing is applied quite regularly, recent analytical investigations have shown that the application of what is commonly assumed to be moderate filtering of shock data can have significant effects on the response frequency and amplitude (Gaberson, 2010). As such, a brief investigation has been performed to justify the use of such filtering techniques for the data acquired in this study.

In the case of accelerometers mounted to the slabs, it was found that the measured responses primarily consisted of two parts: an initial transient response which was comprised of high amplitude and high frequency oscillations, and a stabilized response which possessed lower frequency content and smaller amplitudes. As illustrated in Figure 4.60, peak transverse slab accelerations were found to occur prior to the measurement of any significant transverse displacements occurring at the same location. From the data, it is not clear what caused the lag between initial accelerations and initial displacements; however, this effect was observed routinely. As such, in the case of the slab acceleration response, preservation of peak amplitude within the initial transient response may be of less concern in determining appropriate filter

settings. Note that the potentiometers used in this study possess relatively low resolution capabilities, and subtle high frequency vibrations due to an initial shock front likely would not have been captured in the displacement responses.



The measured response from accelerometer A5 during impact event TH4-1 has been considered as a representative slab acceleration measurement. Based on the determined frequency content of this response (see Section 4.3.4), it is anticipated that low-pass filtering of frequencies greater than approximately 12 kHz will have only minor effect on the overall signal. In Figure 4.61, the measured acceleration response from A5 has been processed using low-pass filters with cutoff frequencies ranging from 1.2 kHz to 12 kHz. The selected cutoff frequencies span the typically recommended range of 10 to 15 % of the bandwidth (Found et al, 1998), where bandwidth refers to one half of the sampling rate. The low-pass filters were designed as tenth-order Butterworth filters and applied using the '*filtfilt*' command provided within MATLAB's predefined signal processing toolbox. Note that the '*filtfilt*' filtering algorithm is a forward-backward filtering algorithm which filters the input signal in forward time and then in reverse time. The forward-backward filtering method is advantageous over other methods due to the reverse pass which results in a time history response with zero phase delay (MathWorks, 2009). The zero phase delay characteristic is critical for analyzing and comparing time history data sampled from multiple sensors.



Figure 4.61 – Filtered Slab Accelerations; A5 TH4-1 ($\rho_l = 0.420$ %; $V_f = 1.00$ %; m = 150 kg)

From the results presented in Figure 4.61, it is apparent that the filters with cutoff frequencies of 9.6 kHz and 12 kHz had little effect on the signal. Application of filters with cutoff frequencies of 4.8 kHz, 2.4 kHz and 1.2 kHz, showed progressive reductions in terms of high frequency contents, amplitudes, and peak response impulses (denoted as I). The filtered time history responses resulting from the 2.4 kHz and 1.2 kHz cutoffs proved to be most successful in filtering out high frequency oscillations while still reasonably preserving the measured

acceleration response. Reductions in the computed peak impulse values are believed to be acceptable with the maximum reduction being approximately 5 %. It is worth re-iterating that this level of low-pass filtering (i.e., less than 3 kHz cutoff frequency) is what has been typically done by previous researchers studying impact phenomena. The same set of filters was applied to the signal from one of the drop-weight accelerometers, A13, measured during impact event TH4-1. The results are presented in Figure 4.62.



Figure 4.62 – Filtered Mass Accelerations; A13 TH4-1 ($\rho_l = 0.420$ %; $V_f = 1.00$ %; m = 150 kg)

The filtered time histories from the drop-weight accelerations show that only minor reductions of peak amplitude were observed from low-pass filtering with cutoff frequencies greater than or equal to 7.2 kHz. The use of the 1.2 kHz cutoff frequency (see Figure 4.62f) was successful in removing most of the post-peak high frequency components and resulted in an impulse reduction of less than 2 %. Remaining high frequency contributions within the 1.2 kHz-filtered response minimally affect the initial impact phase of the response and, as a result, are of less concern.

Evaluating appropriate filter settings for accelerometers from a single impact event is challenging. Other researchers performing drop-weight impact investigations have come to similar conclusions, finding that filter demands can vary within a single testing program due to variables such as impact condition and specimen damage (Found et al., 1998). Based upon the presented filtered acceleration-time history signals, it is evident that no decisive conclusions can be drawn regarding appropriate filter settings which are applicable for all accelerometers used, and all tests performed, in this experimental program. However, the resulting time history responses generated using the 1.2 kHz and 2.4 kHz cutoff frequency filters were most effective in removing the high frequency oscillations and only minimally impacted the computed impulses. As such, the use of the 1.2 kHz frequency cutoff will be used for filtering acceleration data presented in the subsequent sections of the thesis.

Measured signals from the load cells have been investigated to determine if the responses would benefit from some level of filtering. The North load cell from impact event TH6-1 was selected as a representative signal since it displayed the highest frequency content of those previously investigated. Given that the load cells possessed significantly lower frequency contents than that of the accelerometers, filter cutoff frequencies of 1.2 and 2.4 kHz were considered. The filtered time history results for the North load cell are presented in Figure 4.63.

In agreement with the developed frequency response spectrums, the measured load cell responses possessed no significant high frequency contributions, and as a result, low-pass filtering had little effect on the time history response from the North load cell. With the application of the 2.4 kHz cutoff frequency filter, the measured response was unchanged. The response from the 1.2 kHz cutoff frequency resulted in slight smoothing of the peak load plateau.



From the investigation of the signal responses, the following conclusions can be drawn:

- Data pertaining to slab displacements, measured reactions forces from load cells, and strain measurements from the slab reinforcing bars were captured effectively and can be confidently used to investigate the slab behaviours exhibited during the impact tests.
- Data pertaining to the strain measurements of the Dywidag bars used in the testing frame were measured reasonably well. The generated response spectrums suggest that all dominant contributions to the response were captured; however, an increased sampling rate may have resulted in more accurate representations of the peak plateau regions.
- For many of the tests, valid drop-weight acceleration measurements were only obtained for the initial impact. After initial impact, the acceleration responses often saturated due to high frequency vibrations occurring within the steel drop-weight, voiding the impact forcetime histories.
- Response power spectrums developed for selected accelerometers suggest that all significant frequency contributions were captured with the 96 kHz sampling rate. It was determined that typically used signal filtering levels are indeed appropriate.

4.4 Displacements and Deformations

This section of the thesis provides discussion of the displacement behaviours exhibited by the slabs under impact. Displacement-time histories from selected impact events are presented, and are used to illustrate typical responses observed throughout the testing program. Deformed

shapes of the slabs under impact are presented and their relevance is discussed. The midpoint displacement-time histories from all impact tests have been included in Appendix C.

4.4.1 Midpoint Displacement-Time History

The midspan-, or midpoint-, displacement response is perhaps the single-most telling piece of data obtained from experimental investigations of simple beam and slab type elements under impact. Assumptions that overall element behaviour can be directly correlated with that of a single point are often used to perform analytical investigations. This type of investigation is referred to as a single-degree-of-freedom (SDOF) analysis, and in the case of beams and slabs with simple support conditions, the midpoint is usually selected as the member representative degree-of-freedom. However, owing to the highly nonlinear and non-uniform displacement behaviours exhibited by reinforced concrete elements under impact, the displacement-time history of a single point typically does not adequately represent overall element behaviour (Saatci and Vecchio, 2009). As such, care should be taken in assessing element response, either experimentally or analytically, using a SDOF type of approach.

The measured event midpoint displacement-time histories from each of the three impact tests performed on TH2 are presented in Figure 4.64. Note that the measurements obtained from potentiometer P1 (see Section 3.4.3) were used to develop the midpoint displacement-time histories for all slabs; however, in cases where P1 sustained damage or detached from the slab, potentiometer P2 was considered. Additionally, note that the midpoint displacement responses do not consider the accumulation of residual midpoint displacements from prior impacts.

The accumulation of damage resulting from the three impacts performed on TH2 is evident from the measured midpoint displacements. Peak displacement amplitudes occurring in the first quarter cycle of the response were 13.2 mm, 18.7 mm, and 26.6 mm, and midpoint residual displacements at the end of each test increased by 2.7 mm, 3.0 mm, and 8.9 mm. Additionally, from Figure 4.64d, period elongation due to the decreasing slab stiffness can be observed.

The displacement-time histories presented for TH2 was found to be typical of that measured for the conventional RC slabs (slabs TH1, TH2, TH6, and TH7). In all cases, the RC slabs exhibited progressively increasing event deformations from one impact to the next, with substantial

increases in the peak displacement amplitude. In the case of the R/FRC slabs (slabs TH3, TH4, TH5, and TH8), the midpoint displacement-time histories were markedly different. Consider the time history responses from slab TH4 presented in Figure 4.65, and note that TH4 is nominally identical to slab TH2, but was constructed using SFRC with a fibre volume fraction of 1.00 %.



Figure 4.64 – TH2 Midpoint Displacement-Time History ($\rho_l = 0.420$ %; $V_f = 0$)

For comparative purposes, the displacement responses from TH2 have been included in Figure 4.65, and for clarity, intermediate impacts TH4-4 and TH4-6 have not been presented. It can be seen that the addition of the steel fibres led to significant reductions of the peak and residual displacements, particularly for the impacts performed on the previously-damaged slabs (i.e., second impacts, third impacts, etc.). It is also interesting to note that after the first impact event which initially cracked slab TH4, only marginal increases in the residual displacements occurred as a result of the second and third impact events. The increase in post-cracking strength and resistance to permanent deformations was observed for all of the R/FRC slabs.



Figure 4.65 – TH4 Midpoint Displacement-Time History ($\rho_l = 0.420$ %; $V_f = 1.00$ %)

The peak and residual displacements pertaining to each impact event have been summarized in Table 4.5. Note that the total displacement values presented in the table include the accumulation of the residual displacements from prior impacts.

The midpoint displacement characteristics of slabs TH2, TH3, TH4, and TH5 have been presented in Figure 4.66 to investigate the influence of the fibre volume on the observed

displacement behaviours. Note that all of the slabs presented in Figure 4.66 contained the same amount of conventional longitudinal reinforcement.

Event	Event Disp (mm)		<u>Total Disp (mm)</u>		Event	Event Disp (mm)		Total Disp (mm)	
	Peak	Resid.	Peak	Resid.	Lvent	Resid.	Peak	Resid.	Peak
TH1-1	-	-	-	-	TH5-4	22.2	2.8	27.8	8.5
TH1-2	18.8	4.3	-	-	TH5-5	22.9	2.8	31.4	11.3
TH1-3	22.4	7.8	-	-	TH5-6	25.7	4.0	37.0	15.3
TH2-1	13.2	2.7	13.2	2.7	TH5-7	26.3	4.0	41.7	19.3
TH2-2	18.7	3.0	21.4	5.7	TH5-8	29.1	5.4	48.4	24.7
TH2-3	26.6	8.9	32.3	14.7	TH5-9	29.4	4.4	54.2	29.2
TH3-1	-	2.9	-	2.9	TH5-10	29.8	4.2	59.0	33.4
TH3-2	17.3	1.0	20.3	3.9	TH6-1	18.3	6.2	18.3	6.2
TH3-3	22.0	3.3	25.9	7.2	TH6-2	27.7	11.4	33.9	17.6
TH3-4	27.3	7.1	34.5	14.3	TH7-1	12.8	2.6	12.8	2.6
TH3-5	32.5	13.3	46.8	27.6	TH7-2	18.3	3.3	20.9	5.9
TH4-1	14.0	2.7	14.0	2.7	TH7-3	28.5	11.5	34.4	17.4
TH4-2	17.0	1.0	19.7	3.7	TH8-1	13.5	2.7	13.5	2.7
TH4-3	21.6	2.2	25.2	5.9	TH8-2	17.0	0.7	19.7	3.4
TH4-4	24.1	3.7	30.0	9.5	TH8-3	20.4	1.0	23.7	4.4
TH4-5	25.9	4.6	35.4	14.2	TH8-4	23.7	2.5	28.1	6.9
TH4-6	29.2	6.9	43.4	21.1	TH8-5	25.0	3.2	31.9	10.0
TH4-7	32.0	9.1	53.1	30.2	TH8-6	27.3	4.4	37.3	14.5
TH5-1	12.4	3.4	12.4	3.4	TH8-7*	27.5	5.2	42.0	19.7
TH5-2	15.2	0.6	18.7	4.0	TH8-8	31.5	8.8	51.2	28.4
TH5-3	19.3	1.7	23.3	5.7					

Table 4.5 – Midpoint Displacement Characteristics

* based on measurements from potentiometer P6




As illustrated in Figure 4.66, the fibre volume fraction had a significant influence on the midpoint displacement responses of the slabs. R/FRC slabs constructed with higher fibre contents exhibited increased post-cracking stiffness and strength. In comparison to the RC slabs, the R/FRC slabs exhibited smaller displacement amplitudes under like impacts, and were able to withstand additional impacts at higher mass levels prior to failure.

The influence of the longitudinal reinforcement ratio on the displacement responses of the slabs is presented in Figure 4.67. It can be seen that some apparent increase in slab stiffness is realized as the steel reinforcement ratio increased from 0.273 % to 0.420 %. However, in terms of the displacement responses, no additional benefits were realized from increasing the longitudinal reinforcement ratio from 0.420 % to 0.592 %.



Figure 4.67 - Influence of Longitudinal Reinforcement Ratio on Midpoint Displacement

4.4.2 Displaced Shapes

As noted above, investigation of the midpoint displacement-time history alone is not adequate in assessing the global displacement behaviour of elements under impact loading conditions. Due to the high rate of loading in impact testing, inertial effects play a significant role in the displacement behaviour and lead to deformed shapes that may substantially differ from those observed under static loads. Additionally, localized failures and damaged regions are typically more evident from examination of the overall member displacements. Displaced shapes created using potentiometers P3 through P10 (see Figure 4.68) have been presented for slabs TH2, TH5, and TH6.

Note that in some cases displacement measurements from instruments used to develop the deformations over the cross section were not successfully captured due to sensor damage or sensor detachment. In these cases, approximate transverse displacement values based on interpolation of surrounding measurements were used to complete the displacement profiles. All assumed displacement values have been noted in the presented deformed shapes. The displacement profiles are presented over varied time increments to illustrate the progressive deformation of the slabs. For each impact event presented, two plots have been provided: a plot illustrating the progression of slab deformations up to the time of peak midpoint displacement amplitude, and a second plot presenting the deformed shapes of the slab at the time of peak downward (negative) midpoint displacement amplitude, at the time of peak upward (positive) displacement amplitude, and the residual displaced shape of the slab after completion of the test.



Figure 4.68 – Potentiometers Forming Displaced Shapes

The displaced shape of slab TH2 from each of the three impacts performed is presented in Figure 4.69. It can be seen that over the progression of loading, a localized punching behaviour ultimately occurred within the midpoint region. In Figure 4.69c the initial punching formation is somewhat evident from the deformation progression of the slab under the second impact. Under the third and final impact, the punching failure is evident throughout all stages of impact event TH2-3 (see Figure 4.69e and f).



Figure 4.69 – Slab TH2 Deformations ($\rho_l = 0.420$ %; $V_f = 0$)

Similar deformation behaviour was exhibited by RC slab TH6 (see Figure 4.70). Under the initial impact, large deformations occurred but no formation of punching was evident. The second impact resulted in extensive punching throughout the midpoint region of the slab which is apparent from the residual shape of the slab after impact TH6-2 (see Figure 4.70d). The residual and peak deformations of the slab were similar to those exhibited by TH2 during event TH2-3.



Lastly, the cross section deformation behaviour of slab TH5 is presented in Figure 4.71 and Figure 4.72. Note that deformations from only five of the ten impacts performed on TH5 have been presented. Unlike the deformation profiles exhibited by the RC slabs, TH5 did not exhibit signs of punching. The deformed shapes from events TH5-1 through TH5-7 closely resemble an



elastic response, and it is not until event TH5-10 that non-uniform deformation is apparent. The figure also shows the apparent increased deformation capacities achieved by the R/FRC slabs.

Figure 4.71 – Slab TH5 Deformations (Impacts 1, 3, and 5) ($\rho_l = 0.420$ %; $V_f = 1.50$ %)



Figure 4.72 – Slab TH5 Deformations (Impacts 7 and 10) ($\rho_l = 0.420$ %; $V_f = 1.50$ %)

4.5 Loads and Reactions

This section presents the results from the measured support reaction forces and applied impact loads. The calculation of the total reaction forces measured during each impact event is discussed and typical results are provided. Support reaction force-time histories pertaining to all impact tests have been provided in Appendix C.

Selected impact load results are also included in this section. Typical impact force-time histories are presented and are used to evaluate impulse magnitudes. Lastly, force-time histories are used in conjunction with measured slab acceleration data to investigate force equilibrium.

4.5.1 Support Reactions

The reaction forces of the system were measured at each corner of the slab using a combination of load cells and instrumented tie-down assemblies. The force responses from the four corner supports were typically found to be similar in terms of magnitude and response time (see Figure 4.73 and Figure 4.74). It should be noted that in some cases, developed compressive forces in the tie-down assemblies were calculated to exceed the initial post-tensioning forces applied to the Dywidag rods. It is believed this behaviour was the result of inertial forces developed from the mass of the tie-down assembly pushing the Dywidag rods into compression during the tensile unloading phase of the rods. As such, calculated compressive forces exceeding initial post-tension values were corrected in the tie-down force responses. For the majority of the impact tests, the tie-down force correction had only a marginal effect or was not required.





Figure 4.74 – Tie-down Forces; Event TH4-2 ($\rho_l = 0.420$ %; $V_f = 1.00$ %; m = 180 kg)

The reaction forces from the four corner supports were combined to evaluate the overall slab reaction force-time history. The total reaction force-time history from impact event TH4-2 is presented in Figure 4.75. Recall that the tie-down force measurements and the load cell force measurements were collected using independent data acquisition systems. The responses presented in Figure 4.75 have been aligned, post-test, using the signal from potentiometer P20 which was acquired using both data acquisition systems.



Figure 4.75 – Total Support Reaction; Event TH4-2 ($\rho_l = 0.420$ %; $V_f = 1.00$ %; m = 180 kg)

As a means of presenting typical reaction force-time histories observed throughout the testing program, total reactions measured from slab TH4 and TH7 are presented in Figure 4.76 and Figure 4.77, respectively. From the figure below, it can be seen that the magnitude of the peak reaction remained essentially constant throughout the testing of the R/FRC slab. Additionally, from Figure 4.76f, longer reaction response times and increased periods are observed over the progression of impacts imparted to slab TH4.



Figure 4.76 – TH4 Support Reaction-Time Histories ($\rho_l = 0.420$ %; $V_f = 1.00$ %)

In the case of RC slab TH7, it can be seen that a significant decrease in the magnitude of the support reaction was measured under the third and final impact (see Figure 4.77c). This behaviour was exhibited by all slabs, RC and R/FRC, that were subjected to impacts after significant levels of localized punching had already occurred (e.g., in slab TH7 punching had occurred previously in event TH7-2).

The peak reaction forces pertaining to each impact are presented in Table 4.6.



Figure 4.77 – TH7 Support Reaction-Time Histories ($\rho_l = 0.592$ %; $V_f = 0$)

Impact	Reaction	Impact Reaction		Impact	Reaction	
Event	(kN)	Event	Event (kN)		(kN)	
TH1-1	510	TH4-4	789	TH6-1	483	
TH1-2	608	TH4-5	701	TH6-2	458	
TH1-3	422	TH4-6	643	TH7-1	605	
TH2-1	556	TH4-7	591	TH7-2	588	
TH2-2	676	TH5-1	531	TH7-3	368	
TH2-3	580	TH5-2	725	TH8-1	621	
TH3-1	523	TH5-3	779	TH8-2	721	
TH3-2	684	TH5-4	756	TH8-3	759	
TH3-3	743	TH5-5	703	TH8-4	772	
TH3-4	637	TH5-6	777	TH8-5	720	
TH3-5	401	TH5-7	755	TH8-6	767	
TH4-1	591	TH5-8	766	TH8-7	692	
TH4-2	724	TH5-9	759	TH8-8	684	
TH4-3	703	TH5-10	739			

Table $4.6 -$	Peak	Support	Reactions
		~~~~~	1

To assess the influences of the steel fibre volume fraction and the longitudinal reinforcement ratio, peak support reactions from each impact event have been plotted with respect to the applied impact mass in Figure 4.78 and Figure 4.79.

From the data presented in Figure 4.78, it is clearly shown that the addition of the steel fibres resulted in substantial increases in post-cracking strength. Additionally, the slabs' abilities to maintain appreciable post-cracking stiffness correlates with the fibre volume fraction (see Figure 4.78a). Lastly, the addition of fibres resulted in increased support reaction magnitudes; however, measured peak support reactions were found to be similar for all fibre volume fractions considered in the experimental program.

The peak reaction forces have been summarized in Figure 4.79 for the slabs without steel fibres. As was observed from the displacement behaviours, no discernible trend regarding the influence of the longitudinal reinforcement ratio could be established for the RC slabs. Note that slab TH1 has not been included in the figure since a different loading protocol was employed.



Figure 4.78 - Influence of Fibre Volume Fraction on Peak Reaction Force



Figure 4.79 – Influence of Longitudinal Reinforcement Ratio on Peak Reaction Force

# 4.5.2 Impact Loads

Impact force-time histories were calculated using acceleration measurements of the impacting drop-weight. This method of measuring impact force is commonly used in drop-weight impact testing programs but, as noted in Section 4.3, can prove challenging for numerous reasons.

To corroborate the suitability of using the measured drop-weight accelerations to calculate impact forces, trial test drops were performed in which the drop-weight directly impacted the four load cells comprising the corner supports of the test frame (see Figure 4.80). A 120 kg weight was dropped from a height of 125 mm, and the measured load cell reactions were compared with calculated impact force.

The results from the trial impact test are presented in Figure 4.81. It can be seen that after applying low-pass filtering, the mass accelerations are in good agreement with the measured load cell reactions. The results also further corroborate the proposed filter settings determined in Section 4.3. Although a relatively low velocity impact condition was considered in the trial test, it was felt that the impact condition between the steel drop-weight and the steel load cell assembly would be suitable in simulating the hard-impact conditions experienced in the testing program.



Figure 4.80 – Trial Drop-Weight Impact



Figure 4.81 - Corroboration of Impact Force Measurement

The impact force-time histories from selected impact events are presented for slabs TH2 and TH5. The impact events from these slabs comprise a range of slab stiffnesses and impact mass levels used in the experimental program.

The drop-weight accelerometers were filtered using a low-pass Butterworth filter with a 1.2 kHz cutoff frequency as discussed in Section 4.3. However, even after filtering, many of the impact responses contained high frequency contributions. Impact force-time histories were only calculated for impact events in which at least one of the two accelerometers on the drop-weight measured a response free from saturation for the entire event. The calculated peak impact forces, noted as  $F_{max}$ , and calculated response impulse values, noted as I, have been included on each time history plot. Trailing nonzero values were not included in the event impulse calculations.

The impact force responses from two impact events performed on slab TH2 are presented in Figure 4.82. In the case of event TH2-2, the time history response was calculated using a single accelerometer as opposed to an averaged value; as such, the peak impact force of 1,455 kN may not be representative. In comparing the impact responses from the two events below, it is interesting to note that the event impulse was determined to be significantly larger for TH2-3 even though event TH2-2 exhibited a higher peak impact force.

The time history responses from four of the ten impact events performed on slab TH5 are presented in Figure 4.83. It can be seen that through the progression of tests performed on TH5, the peak impact force changed very little; however, the impulse was found to increase

substantially as the drop-weight mass and the slab damage increased. Additionally, the shape of the impact force response also changed over the course of the impacts with a secondary peak becoming more prominent as the slab damage increased.



Figure 4.83 – Impact Force-Time History; Slab TH5 ( $\rho_l = 0.420$  %;  $V_f = 1.50$  %)

Based on the results presented in Figure 4.82 and Figure 4.83, it can be seen that the peak impact force does not seem to provide a meaningful indication of impact severity. Rather, the use of the event impulse from the impact force-time history is likely more appropriate.

## 4.5.3 Dynamic Equilibrium

As noted in Section 3.4.1, one quadrant of the slab was densely instrumented with nine accelerometers in an effort to obtain an accurate representation of the transverse slab accelerations. With the assumption that the acceleration behaviour was relatively symmetric throughout the four quadrants of the slab, the inertial force of the slab was calculated using numerical integration of the product of slab accelerations and tributary slab masses. Parabolic shape functions (reported in Section 5.1.3) were used to estimate the distribution of the slab accelerations, and the method of Gauss with a 3 x 3 integration scheme was used to perform the numerical integration.

Estimated distributions of the measured slab accelerations from impact event TH5-1 are presented in Figure 4.84. It can be seen that the computed acceleration distributions are highly non-uniform and change significantly over the course of the impact event. As such, it is evident that the distribution of the forces acting on the slab also vary over the course of the event and that inertial force development resulted in loading distributions which are significantly different from those developed under conventional monotonic/static testing conditions.

To investigate the interaction of the forces occurring throughout the impact events, the responsetime histories of all force components from event TH5-1 have been presented in Figure 4.85. The figure includes the measured impact force from the drop-weight, determined support reactions measured from the load cells and Dywidag tie-down assemblies, and the slab inertial forces computed from estimated slab acceleration distributions.

From Figure 4.85, it can be seen that under the highly impulsive loading conditions considered in this test program, the inertial forces of the slab played a dominant role in resisting the applied impact force. In the presented force-time history behaviour for impact event TH5-1, it can be seen that the peak impact force is almost entirely resisted by the inertial forces of the slab.



Figure 4.85 – Dynamic Equilibrium; Event TH5-1 ( $\rho_l = 0.420$  %;  $V_f = 1.50$  %; m = 150 kg)

The computed slab inertial force response was found to counterbalance the applied impact force with a reasonable level of accuracy; however, force equilibrium with the slab support reactions immediately after mass impact was not represented well. The post-impact response (i.e., after the initial 15 ms) is equilibrated well, with slab inertial forces counterbalancing the support reactions in magnitude and phase. As such, it is evident that the accelerometer instrumentation scheme used to develop the inertial responses of the slab resulted in only limited success, and findings attributed to the calculated inertial force should be treated with caution.

## 4.6 Impact Energy

The capacities of the slabs under the prescribed impact loading protocol have been computed on the bases of the total kinetic energy imparted ( $E_{imp} = \frac{1}{2} mv^2$ ) to each slab. Recall that the testing of each slab was terminated when any one of the following criteria was satisfied: i) the full tenimpact loading protocol was completed (see Table 3.5), ii) the measured support reaction forces decreased significantly from those measured during the previous impact event, or iii) the damaged state of the slab greatly increased the likelihood of damaging the instrumentation under an additional impact. As such, the impact energy results presented in this section should not be treated as absolute energy capacities, but rather as relative capacities for a series of slabs subjected to a common loading protocol and governed by a common set of failure/termination criteria.

The total imparted energy and the governing test termination criteria for each of the slabs are summarized in Table 4.7. Recall that completion of the full ten-impact testing protocol governed testing termination of slab TH5 and, as such, the total imparted energy computed for the slab should be considered as a lower-bound estimate of the of the impact energy capacity.

The influence of the longitudinal reinforcement ratio and influence of the steel fibre volume fraction on the slab impact energy capacity is presented in Figure 4.86. From the figure it can be seen that increasing the longitudinal reinforcement ratio had limited influence on the capacities of the RC slabs, with equal capacities being obtained for Slabs TH2 ( $\rho_l = 0.420$  %) and TH7 ( $\rho_l = 0.592$  %). In Figure 4.86b it is evident that the addition of the steel fibres were highly effective in increasing the impact capacities of the slabs, and that the increased strengths of the R/FRC slabs tended to correlate with the steel fibre volume fraction. Similar strength trends

attributed to the steel fibre volume fraction were obtained for slabs containing 0.420 % (slabs TH2, TH3, TH4, TH5) and slabs containing 0.592 % (slabs TH7 and TH8) longitudinal reinforcement.

Table 4.7 – Imparted Energy

Slab	$egin{array}{c}  ho_l \ (\%) \end{array}$	V _f (%)	E _{imp} (kJ)	Termination Criteria *
TH1	0.420	-	14.4	2, 3
TH2	0.420	-	17.3	2, 3
TH3	0.420	0.50	32.6	2, 3
TH4	0.420	1.00	49.9	2, 3
TH5	0.420	1.50	78.7	1
TH6	0.273	-	10.6	3
TH7	0.592	-	17.3	2, 3
TH8	0.592	1.00	59.5	3

1: completed testing protocol (see Table 3.5)

2: decreased support reaction measurements

3: extensive slab damage



In comparing Figure 4.86a and Figure 4.86b, it can be seen that the R/FRC slabs greatly outperformed the RC slabs in terms of impact energy capacity. The limited strength increases resulting from the increased longitudinal reinforcement ratios can be attributed to the punching shear failure modes which were found to govern the behaviours of the slabs under impact.

#### 4.7 Rebar Strains and Strain Rates

This section presents measured reinforcing bar strain-time histories from selected impact events. Typically, strain gauges located on the bottom mat of reinforcement within the midpoint region measured the largest peak strain values and displayed the largest residual strain levels. As such, strain-time histories measured from gauge S1 have been presented for impact events performed on slabs TH2 and TH5. Note that the strain data provided in the following figures are presented in the forms of both total strains and measured event strains. Total strain measurements include the accumulation of residual strains from prior impacts. Additionally, recall that the monotonic static yield strain for the U.S. #3 bars comprising the longitudinal reinforcement in slabs TH2 and TH5 was determined to be approximately  $2.5 \times 10^{-3}$  mm/mm.

From the strain behaviours presented in Figure 4.87, it can be seen that strain levels greater than static yield were achieved under all three impact tests performed on TH2. Additionally, the residual strain measured in gauge S1 was found to increase substantially after each impact. The magnitude of the peak event strain measured at the location of gauge S1 decreased over the course of testing TH2. This is most likely the result of localized strains at the location of the circumferential punching crack noted in the test observations for slab TH2 (see Section 4.2), which were found to occur outside of the midpoint impact region.

The strain-time history for slab TH5 is presented in Figure 4.88. After the first impact (test TH5-1) little change in the measured strain behaviours from impacts TH5-2 to TH5-7 was observed, with peak total strains remaining in the order of 4.0  $\times 10^{-3}$  mm/mm and total residual strains recorded in the range of 0.40  $\times 10^{-3}$  to 0.60  $\times 10^{-3}$  mm/mm. Impacts performed using the 300 kg drop-weight (events TH5-8 through TH5-10) resulted in significant bar straining with increases in both measured peak and residual strains. With the tenth and final impact test performed on TH5, a peak total strain of 9.6  $\times 10^{-3}$  mm/mm and a residual strain value of 6.2  $\times 10^{-3}$  mm/mm were recorded for strain gauge S1. In contrast to the reinforcing bar strains exhibited by slab TH2, the bar strains in slab TH5 increased significantly over the course of the final impacts, indicating that the strains had not localized outside of the midpoint region.



Figure 4.87 – TH2 Strain-Time History; Gauge S1 ( $\rho_l = 0.420$  %;  $V_f = 0$ )

From the strain data presented for slabs TH2 and TH5, it is evident that the reinforcing bars in the slabs regularly experienced strain levels which exceeded the quasi-static yield strain. Additionally, the presence of large residual strains suggests that the reinforcing bars did incur permanent deformations as a result of steel yielding. However, it should be noted that the degree of yielding, and the stresses developed in the reinforcing bars, may have been significantly impacted by the high strain rates which were found to occur throughout testing.



Figure 4.88 – TH5 Strain-Time History; Gauge S1 ( $\rho_l = 0.420$  %;  $V_f = 1.50$  %)

The measured reinforcing bar strain rates from four impact events are presented in Figure 4.89. It can be seen that in all events considered, strain rates between 1.0 s⁻¹ and 10 s⁻¹ were found to occur during initial mass impact. After the initial impact, rates in the order of 0.01 s⁻¹ to 0.15 s⁻¹

occurred regularly throughout the remainder of the impact response. It has been shown in numerous studies that deformations occurring at strain rates exceeding  $0.01 \text{ s}^{-1}$  may lead to significant increases in the apparent tensile yield stress and marginal increases in the apparent ultimate stress (Asprone et al., 2009; CEB, 1988; Malvar, 1998). Deformations occurring at rates greater than  $1.0 \text{ s}^{-1}$  significantly impact both yield and ultimate tensile stresses.



Figure 4.89 – Measured Strain Rates

#### 4.8 Damping Characteristics

To characterize the damping behaviours exhibited by the slabs throughout the testing program, a number of forced free vibration displacement tests were used to estimate the damping characteristics. For most slabs, prior to performing each impact test, a rubber mallet was used to

strike the slab at the midpoint location. The blow from the rubber mallet resulted in low amplitude free vibrations which were captured in the displacement-time history responses.

The measured periodic decay in the displacement responses were used to estimate the damping ratios of the slabs,  $\xi$ , using the logarithmic decrement based approach presented in Equations 4-7 and 4-8 (Paultre, 2011):

$$\frac{\xi}{\sqrt{1-\xi^2}} = \frac{1}{2m\pi} \ln \frac{u_n}{u_{n+m}}$$
(4-7)

$$\xi \approx \frac{1}{2m\pi} \ln \frac{u_n}{u_{n+m}} \tag{4-8}$$

where,

 $\xi$  = ratio of critical damping;

m = number of cycles between peak displacement values  $u_n$  and  $u_{n+m}$ ;

 $u_n$  = displacement amplitude at time  $t_n$ , mm;

 $u_{n+m}$  = displacement amplitude at time  $t_{n+m}$ , mm.

Note that for under-damped displacement responses with low damping ratios, the approximated damping ratio presented in Equation 4-8 is a valid approximation of Equation 4-7.

The estimated damping ratios were verified using developed theoretical peak displacement response envelopes calculated from Equation 4-9 (Paultre, 2011):

$$u^*(t) = u_{\circ}e^{-\xi\omega t} \tag{4-9}$$

where,

 $\omega$  = angular frequency, rad/s;

t = elapsed time, s;

 $u_o$  = initial displacement at t = 0, as illustrated in Figure 4.90, mm.



Figure 4.90 – Damped Free Displacement Response (Paultre, 2011)

The pre-cracking and post-cracking free vibration responses for slabs TH4 and TH6 have been presented in Figure 4.91 and Figure 4.92, respectively. For slab TH4, in the uncracked condition, the damped natural period of the slab,  $T_D$ , was estimated to be approximately 13.7 ms, and the apparent damping was estimated to be in the order of 1.7 % of critical. After the slab had undergone multiple impact tests, the period of the slab increased to approximately 28.4 ms, and the apparent damping was estimated to be 2.3 % of critical.



Similar results were found for slab TH6 (see Figure 4.92). In the uncracked condition, the slab's damped natural period was approximately 12.8 ms, and the apparent damping of the slab was estimated to be 1.5 % of critical. After a single impact event (i.e., after TH6-1, before TH6-2),



the natural period increased to approximately 27.0 ms and the apparent damping increased to approximately 2.3 % of critical.

From the figures presented, it can be seen that the use of logarithmic decrement approach resulted in reasonable estimates of the apparent damping ratios which is evident from the good agreement between the theoretical response envelopes calculated from Equation 4-9 and the measured free vibration responses. It should be noted that given the low amplitudes of the free vibration responses, the displacement-time histories from these tests could not be captured with high resolution.

In the uncracked condition, the apparent damping ratios were estimated to be in the order of 1.5 to 1.7 % of critical for slabs TH4 and TH6, respectively. After sustaining appreciable damage, the apparent damping ratios increased to 2.3 % of critical for both slabs.

## 4.9 Slab Damage

Static analyses of the slabs tested in the experimental program were performed using the nonlinear finite element analysis program VecTor4. Details pertaining to the slab modelling and the analytical results under monotonic loading conditions are discussed in Chapter 7. In brief, the analytical results suggest that under static loading conditions, all of the slabs in the program were controlled by flexural failure mechanisms. For all reinforcement levels considered, the slabs

were predicted to undergo extensive tension steel yielding and achieve high levels of ductility prior to failing.

As documented in the impact test observations (see Section 4.2), the presence of wide circumferential cracking indicative of punching shear was noted for all slabs with the exception of slab TH5 which was comprised of SFRC with 1.50 % fibre volume fraction. It is important to note that the analytical static capacities of the slabs were controlled by highly ductile flexural behaviours, and the observed damage of the slabs under impact loading showed that the majority of the slabs were in fact controlled by shear failures at the location of impact.

The crack pattern development for slab TH2 is presented in Figure 4.93. Recall that slab TH2 was constructed using conventional concrete, and contained no out-of-plane shear reinforcement. Under the initial impact (event TH2-1), the widest crack was measured to be 0.50 mm and was observed on the bottom surface of the slab, occurring circumferentially around the impact region. No mass penetration or concrete scabbing was observed under the initial impact. In the subsequent impacts, TH2-2 and TH2-3, damage indicative of punching shear developed at the location of the circumferential crack. The shear failure was coupled with extensive concrete scabbing and mass penetration which was measured to be approximately 20 mm in depth.

The damage and cracking behaviour of slab TH2 was found to be typical of that observed for all of the conventional RC slabs. Under initial impact, the formation of a large circumferential crack on the bottom surface was found to occur, and little or no damage in the form of scabbing and penetration was observed. Subsequent impacts resulted in progression of the already developed punching region, and extensive concrete scabbing and mass penetrations were observed. Crack orientation and spacing closely matched the longitudinal reinforcement configuration.













Figure 4.93 – TH2 Crack Development ( $\rho_l = 0.420$  %;  $V_f = 0$ )

In the case of the R/FRC slabs, the evolution of damage tended to progress more toward flexural damage characteristics as the fibre volume fraction increased. That is, as the fibre levels increased, the severity and the development of the localized shearing behaviour within the impact region was found to become less significant. Under initial impacts, the formation of circumferential cracks was still observed; however, the cracks were better controlled with smaller spacings and widths which ranged from 0.10 to 0.15 mm. The largest cracks were radially oriented on the bottom surfaces of the slabs, with small widths ranging from 0.15 to 0.20 mm.

In the case of slabs TH3 and TH4 which contained 0.50 % and 1.00 % fibre volume fractions, respectively, the punching shear development was not apparent until after the third impact event was performed. Subsequent impacts led to more pronounced punching behaviours with increasing levels of mass penetration. Due to the increased fibre content, slab TH4 required more impacts to achieve a level of damage similar to that observed in slab TH3. Ultimately, the mass penetration for TH3 was measured to be approximately 25 mm after five impacts events, and penetration for TH4 was approximately 18 mm after seven impact events. Concrete scabbing was limited to the main circumferential crack, and was similar for both slabs.

Slab TH5 contained the highest fibre content with a volume fraction of 1.50 %; as a result, the evolution of damage observed in TH5 was markedly different than all other slabs in the experimental program. The crack pattern development on the bottom surface of slab TH5 is presented in Figure 4.94. In comparison to the cracking behaviour observed for TH2, the addition of the steel fibres led to a significant reduction in the apparent crack spacing, even under initial impacts. Additionally, it is evident from the figure that no concrete scabbing was observed to occur as a result of any of the ten impact events imposed on slab TH5.



Figure 4.94 – TH5 Crack Development; *bottom surface* ( $\rho_l = 0.420$  %;  $V_f = 1.50$  %)

After impact event TH5-3, a maximum crack width of 0.45 mm was noted to have occurred within a circumferential crack located under the impact region, indicating the potential development of punching. However, unlike slabs TH3 and TH4, the circumferential crack width growth was effectively controlled by the steel fibres, and widths of the radial cracks were found to increase more significantly under subsequent impacts. Mass penetration was first noted to occur after the second impact; however, as was the case with circumferential cracking, penetration development was controlled throughout the testing of slab TH5 with only marginal increases in penetration depth occurring from subsequent impacts. After the fifth impact, the mass penetration was limited to approximately 3.5 to 4.0 mm, and the maximum measured crack width was measured to be 0.90 mm, occurring in a radial crack outside of the impact region. After the seventh impact, mass penetration increased to 5.0 mm and the maximum radial crack width was 1.60 mm. Recall that slab TH3 and TH4 had exhibited extensive localized shearing behaviour after five and seven impacts, respectively.

Slab TH5 was subjected to ten impacts events in total. Testing of the slab was ultimately terminated when it became apparent that the longitudinal steel had extensively yielded (see Section 4.7), and the slab damage became primarily focused on widening of existing radial cracks. After the tenth and final impact, the maximum radial crack width on the bottom surface was measured to be 3.5 mm, and the occurrence of several radial cracks with widths greater than 2.0 mm were noted. The mass penetration was limited to a depth of 6.5 mm, occurring locally under the impacting mass; no dominant punching crack was evident, and no concrete scabbing occurred.

Figure 4.95 presents the final bottom surface cracking patterns for slabs TH2 through TH5. Evident from the figure are an apparent reduction in crack spacing, concrete scabbing, and localized punching as a result of the increased fibre levels. It should be noted that in the case of R/FRC slabs TH3 and TH4, some minimal concrete delamination occurred but is not visible in the figure as the steel fibres prevented the concrete from fully scabbing from the bottom surfaces of the slabs. For slab TH5, the concrete at the bottom surface of the slab remained sound throughout testing; no scabbing or delamination was observed.



Lastly, a summary of the measured crack widths and mass penetration is provided in Table 4.8. The table presents the maximum cracks widths on the top and bottom surfaces of the slabs. Note that crack widths were not measured for all impact events.

	Bottom		Тор			Bottom		Тор	
	$W_{max}$		$W_{max}$	Pen		$W_{max}$		W _{max}	Pen
Event	(mm)	location	(mm)	(mm)	Event	(mm)	location	(mm)	(mm)
TH1-1	0.35	circumf	0.10	0	TH5-1	0.15	radial	0.05	0
TH1-2	1.00	circumf	0.10	1.5	TH5-3	0.45	circumf	0.10	2.5
TH1-3	$2.50^{*}$	circumf	0.10	3.5	TH5-5	0.90	radial	0.10	4.0 ^L
TH2-1	0.50	circumf	0.10	0	TH5-7	1.60	radial	0.15	5.0 ^L
TH2-2	0.50	circumf	0.15	3.5	TH5-10	3.50	radial	0.15	6.5 ^L
TH2-3	$0.60^{*}$	radial	0.20	20	TH6-1	-	-	0.15	3.5
TH3-1	0.20	radial	0.10	0	TH6-2	$0.85^*$	radial	0.30	20
TH3-2	0.50	circumf	0.15	2.0	TH7-1	0.60	circumf	0.15	0
TH3-3	0.80	circumf	0.50	4.5	TH7-2	1.80	circumf	0.15	10
TH3-5	2.00	radial	0.30	25	TH7-3	20	circumf	0.15	30
TH4-1	0.20	radial	0.10	0	TH8-1	0.20	radial	0.10	0
TH4-2	0.35	radial	0.20	1.0	TH8-3	0.20	radial	0.15	1.5
TH4-3	0.90	circumf	0.20	2.5	TH8-5	1.10	circumf	0.15	3.5
TH4-5	1.20	circumf	0.20	6.0	TH8-7	1.10	radial	0.20	12
TH4-7	4.00	circumf	0.60	18	TH8-8	1.40	radial	0.20	20

Table 4.8 – Summary of Measured Slab Damage

* maximum crack width outside of scabbed region

^L occurred locally at the corner of the impact location

Note: Pen = penetration; circumf = circumferential

## 4.10 Test Program Summary

The experimental program was successful in achieving the identified primary objectives: examination of the behaviour of RC and R/FRC slabs under high-mass low-velocity drop-weight impacts, a comparative assessment regarding the influence of the steel fibres on the impact resistance of the concrete slabs, and the acquisition of a well documented data set which is applicable for the development and corroboration of analytical models pertaining to RC and R/FRC elements under extreme loading conditions. Notable conclusions include:

The observed behaviours of the slabs were strongly influenced by shearing effects. Punching shear, or the development of a localized shear plug at the location of impact, controlled the ultimate behaviour of nearly all of the slabs. The observed shear-critical behaviours from the experimental program strongly contrast the ductile flexural failure modes which analytical modelling suggests would occur under quasi-static loading conditions. Investigation of the inertial responses of the slabs showed that non-uniform inertial force distributions resulted in loading conditions which differ from slabs under conventional monotonic/static testing conditions.

- The addition of steel fibres improved the impact responses of the slabs in a number of ways. In comparison to the conventional RC slabs, R/FRC slabs displayed reduced displacement amplitudes under like impacts, and the ability to resist additional impacts of higher mass levels. R/FRC slabs exhibited superior ability in limiting crack widths and reducing crack spacing, and the steel fibres were effective in reducing the severity of slab damage occurring in the form of mass penetration and concrete scabbing. The effectiveness of the steel fibres were highly effective in mitigating the onset of the punching shear failures which ultimately controlled the behaviour of the RC slabs. In the case of slab TH5, which contained a fibre volume fraction of 1.50 %, the fibres entirely prevented punching from occurring.
- An investigation regarding the validity of the digital data was performed. It was determined that the data sampling rates used in the test program were sufficient to capture all significant response contributions. The resolution of the strain-time histories pertaining to the Dywidag bars forming the test frame could have benefitted from an increased sampling rate, specifically to ensure that peak strain values were captured. The data in the experimental program were sampled without the use of digital data filtering algorithms. A brief investigation regarding data filtering showed that no benefit was obtained from post-test filtering of the data obtained from the strain gauges, potentiometers, and load cells. Post-process filtering the acceleration measurements was found to have a significant effect on the measured response, but was deemed necessary to develop meaningful acceleration-time histories.

# **CHAPTER 5: SOFTWARE FORMULATION**

The RC shell finite element analysis subroutines forming the basis of the software program VecTor4 were originally developed in the 1980s by Owen and Figueiras (1984). The original programs were dedicated to the analysis of shell structures and employed nonlinear material models developed for reinforced concrete or elasto-plastic materials. A layered, 'thick-shell,' degenerated shell formulation was considered with the use of a newly developed 'Heterosis' finite element. The programs were general purpose shell analysis programs capable of modelling both thick- and thin-shell structures with consideration of geometric nonlinearity; features which distinguished the programs from most other shell analysis tools available at the time.

In the early 1990s the published source code from the work of Figueiras and Owen (1984) was modified by Polak and Vecchio (1993a) through replacement of the governing behavioural models with that of the constitutive formulations of the Modified Compression Field Theory (MCFT). Additional modifications were made to the program, allowing the consideration of out-of-plane shear reinforcement and the incorporation of new loading capabilities into the analysis subroutines. The resulting program was called APECS.

Seracino (1995) extended the modelling capabilities of program through the implementation of various second-order mechanisms and the improvement of a number of the solution algorithms. Significant revisions regarding the input and output files were made at that time to simplify model creation and to ease interpretation of results. The program was renamed RASP.

It is important to note that the analytical work pertaining to program VecTor4 presented in this thesis was primarily formed from the redevelopment and extension of the works noted above. Although in its present state VecTor4 differs substantially from all of the programs noted above, the formulation of the layered shell finite element and several of the developed subroutines in VecTor4 remain essentially unchanged from that originally considered by Figueiras and Owen.

This chapter provides a brief overview of the shell finite element analysis program VecTor4. A general overview of the assumptions used to facilitate the shell element formulation, as well as references to literature presenting the detailed development of the shell element are provided.

Additionally, the governing RC constitutive behavioural models which have been implemented in the program are briefly described.

# **5.1 Shell Finite Element Formulations**

The following subsections present a brief overview of the layered 'thick-shell' degenerated Heterosis finite element which forms the basis of the analysis program VecTor4. Much of the information regarding the finite element formulation is based on the original presentation provided by Figueiras and Owen (1984); however, its inclusion in the thesis is pertinent in considering the applicability and functionality of the program.

# **5.1.1 Deformation Assumptions**

Most shell finite elements typically fall into one of two categories: i) thin-shell elements developed on the basis of Kirchhoff plate bending theory, ii) and thick-shell elements developed from three-dimensional elasticity in accordance with Mindlin theory, or what is often referred to as Reissner-Mindlin assumptions. The main difference between the two approaches is that elements developed in accordance with Mindlin theory include deformations resulting from out-of-plane shear. With an appropriate numerical integration scheme, the use of Mindlin based elements can be used to model shells exhibiting either thick-shell or thin-shell behaviour (Huang and Hinton, 1986b). This concept is further discussed in Section 5.1.3.

It is not uncommon for reinforced concrete shell structures to experience significant out-of-plane shear forces. Examples of such structures include RC storage silos possessing non-uniform geometries or subjected to eccentric loadings, RC storage containers, and RC offshore structures, to list a few. As such, a thick-shell finite element formulation is more appropriate for the analysis of RC shells.

Shell elements developed using Mindlin theory are isoparametric, displacement type elements based on the following assumptions:

- i) plane sections prior to deformation remain plane after deformation, but not necessarily normal to the element midsurface; and
- ii) stresses normal to the midsurface (i.e., out-of-plane clamping stresses) are negligible.

The first assumption regarding the element deformation is illustrated in Figure 5.1. The total rotations,  $\theta_x$ , occurring at a given section consist of two contributing parts: bending deformations,  $\partial w/\partial x$ ; and shear deformations,  $\phi_x$ . Note that in the figure, w represents the transverse displacement of the examined plane at the element midsurface. The total rotations for a three-dimensional shell element can be generalized using Equation 5-1, where the x and y axes are oriented orthogonally in the plane of the shell.



Figure 5.1 – Shell/Plate Deformation Behaviour (adapted from Polak and Vecchio, 1993a)

#### 5.1.2 Degenerated Shells

Finite element analyses of shell type structures are typically performed using one of three types of finite elements: i) flat elements, which are created through a combination of membrane and bending elements, ii) curved elements, which are developed from the basis of shell theory, and iii) degenerate elements, which are derived on the basis of three-dimensional elasticity. The discussion of the various shell structure modelling approaches provided by Figueiras and Owen (1984) is briefly summarized below.

The use of flat elements to model shell structures has significant limitations. Considering that many shell structures possess curved geometries, extremely fine meshes are required to obtain an accurate representation of curved structures, offsetting the low computational costs of the
simpler flat element. Other problems, such as artificial 'discontinuity' bending moments, may arise when attempting to use flat elements to model curved structure geometry.

Shell analyses employing curved elements are also of limited applicability due to the fact that the elements are formulated directly from shell theory, and typically the various shell theories which have been developed were derived using specific assumptions appropriate for a specific structure under consideration. The nonexistence of a fully generalized shell theory limits the potential applications of using curved shell elements in the development of general finite element software programs.

The use of degenerated elements represents the most generalized approach to modelling shell structures. Because the elements are developed directly from the equations of three-dimensional elasticity, the need for a generalized shell theory is avoided entirely. The use of isoparametric elements which possess independent displacement and rotational degrees of freedom allows three-dimensional stress and strain conditions to be degenerated to shell behaviour. The degeneration from a twenty-node solid element to an eight-node shell with five degrees of freedom per node (u, v, w,  $\theta_1$ ,  $\theta_2$ ) is presented in Figure 5.2.



Figure 5.2 – Degeneration of Three-Dimensional Solid Element (Cook et. al, 1989)
(a) 20-node, 60 degree of freedom (dof) solid element
(b) elimination of four mid-side node yields a 16-node, 48 dof solid element
(c) 8-node, 40 dof shell element

In the original development of the program VecTor4 (originally APECS), it was determined by Polak and Vecchio (1993a) that the generalized formulation method and the ability to consider a wide range of shell type structures made the degenerated shell finite element most suitable for nonlinear analysis of RC shell structures.

### 5.1.3 The Heterosis Shell

The degenerated Heterosis shell is a nine-node quadratic element which has been shown to exhibit superior performance characteristics over the eight-node Serendipity element and the nine-node Lagrangian element, specifically with respect to the analysis of thin-shell structures (Hughes and Liu, 1980). The Heterosis element is developed using Serendipity shape functions for translational degrees of freedom, and Lagrangian shape functions for rotational degrees of freedom (see Figure 5.3). This approach, combined with a selective numerical integration scheme used to avoid 'shear locking' phenomena, results in a shell element which exhibits good performance in both thick- and thin-shell applications, and eliminates the development of spurious zero-energy modes created from the use of reduced integration schemes. In the initial development of the Heterosis plate element, it was determined that removal of the translational degrees of freedom from the central node (denoted as node nine) was required to eliminate all zero-energy modes (Hughes and Cohen, 1976). The developed element results in a shell with a total of 42 dof, with each of the eight side nodes possessing five dof (three translational, and two rotational), and the central ninth node possessing only the two rotational dof.

A summarized comparison of the Heterosis shell element with the Serendipity and Lagrangian elements is provided in Figure 5.3. From the figure it can be seen that the combined formulation used to develop the Heterosis element results in the least number of zero-energy modes, and in meshes with two or more shell elements, the zero-energy modes are eliminated entirely. The same behaviour is found in the Serendipity element; however, even with a reduced integration rule the Serendipity formulation suffers from poor behaviour in thin-shell applications as illustrated by the low constraint index. The constraint index is a heuristic measure developed to quantify an element's ability to accommodate zero shear strain, a prescribed limit requirement for thin-shell elements (Hughes and Cohen, 1976). For quadratic shells, a constraint index of four is considered optimal (the Lagrangian element), and a constraint index of one is considered poor (the Serendipity element). The constraint index of three for the Heterosis element is considered nearly optimal and should be considered acceptable for RC applications where the consideration of out-of-plane shear deformations is pertinent.

	SERENDIPITY	HETEROSIS	LAGRANGE
<i>u</i> , <i>v</i> , <i>w</i> shape functions	Serendipity	Serendipity	Lagrange
$\theta_1, \theta_2$ shape functions	Serendipity	Lagrange	Lagrange
integration scheme	bending: 2 x 2 – Gauss shear: 2 x 2 – Gauss	bending: 3 x 3 – Gauss shear: 2 x 2 – Gauss	bending: 3 x 3 – Gauss shear: 2 x 2 – Gauss
number of spurious zero-energy modes	bending: 1 [*] ¹ membrane: 1 [*]	bending: 0 ¹ membrane: 1*	bending: 1 ¹ membrane: 2+1*
constraint index	1	3	4

¹ from Figueiras and Owen (1984)

* not communicable in a mesh of two or more elements

• -  $u, v, w, \theta_1, \theta_2$  degrees of freedom considered

 $\bigcirc$  -  $\theta_1$ ,  $\theta_2$  degrees of freedom considered

Figure 5.3 – Quadratic Shell Elements (adapted from Hughes and Cohen, 1976)

The ability of the Heterosis element to be generalized such that thick- and thin-shell behaviour can be effectively accommodated relies heavily on the use of a reduced numerical integration scheme. As illustrated above, the Heterosis element was formulated using a selective scheme where a 3 x 3 Gauss integration rule is used to evaluate bending and membrane energy, and out-of-plane shear energy is integrated using 2 x 2 Gauss integration. The selective integration procedure is accommodated through modification of the shear strain responsible terms in the strain-displacement matrix [*B*]. The main steps involved in performing the selective integration procedure are as follows (Figueiras and Owen, 1984):

- Terms in the [B] matrix corresponding to the out-of-plane shear strains are evaluated at the four Gauss points (I, II, III, IV) corresponding to the 2 x 2 reduced integration scheme (see Figure 5.4).
- The [B] matrix values computed in step 1 are extrapolated to the nine Gauss points (1 through 9) corresponding to the sampling points of the normal 3 x 3 integration scheme (see Figure 5.4). Extrapolation is performed using Equation 5-2:

$$B(\xi,\eta)_n = \sum_{r=1}^4 Ns_r(\xi,\eta)_n B(\xi,\eta)_r$$
(5-2)

where  $(\xi,\eta)_n$  and  $(\xi,\eta)_r$  correspond to the Gauss point locations of the normal and reduced integration schemes, respectively. Note that  $\xi$ ,  $\eta$  represent the curvilinear coordinates at the locations of the sampling points. The four shape functions denoted by  $Ns_r$  in Equation 5-2 are defined at the reduced (2 x 2) integration points and, as such, result in unit values at their respective Gauss point, and zero values for the remaining three Gauss points. The shape functions corresponding to the integration points (I, II, III, IV) are defined by Equation 5-3.

$$Ns_{r}(\xi,\eta) = \frac{1}{4}(1+\xi\xi_{r})(1+\eta\eta_{r})$$
(5-3)

In the evaluation of Equations 5-2 and 5-3, the values presented in Table 5.1 should be considered for  $\xi_r$  and  $\eta_r$ .

3) The remaining bending and membrane strain terms required to assemble the [B] matrix are evaluated at the nine Gauss points (1 through 9) corresponding to the normal 3 x 3 integration rule (see Figure 5.4).

-			
Integration Point	Shape Function	ξr	$\eta_r$
Ι	$Ns_1$	-1.0 / -0.57735	-1.0 / -0.57735

 $Ns_2$ 

 $Ns_3$ 

 $Ns_4$ 

Π

III

IV

-1.0 / -0.57735

1.0 / 0.57735

1.0 / 0.57735

1.0/0.57735

-1.0 / -0.57735

1.0 / 0.57735

Table 5.1 – Shape Function Constants for Selective Integration (Figueiras and Owen, 1984)

Having employed the above selective integration modification, the resulting strain-displacement
matrix [B] requires that only sampling points at the nine locations corresponding to the normal
3 x 3 integration rule be considered when computing element stress and strain components.



Figure 5.4 – Gauss Point Locations for Numerical Integration

In the original development of the computer program, a hierarchical formulation was adopted such that all three degenerated elements (Heterosis, Serendipity, and Lagrangian) could easily be incorporated into the programs solution algorithm. Although in its current state program VecTor4 only considers the use of the nine-node Heterosis shell element (as it has been shown to be best suited for RC shells), the original hierarchical formulation is still implemented. The shape functions pertaining to the eight boundary nodes are those of the Serendipity shape functions:

corner nodes: 
$$N_k(\xi,\eta) = \frac{1}{4}(1+\xi\xi_k)(1+\eta\eta_k)(\xi\xi_k+\eta\eta_k-1)$$
(5-4)

midside nodes: 
$$N_k(\xi,\eta) = \frac{\xi_k^2}{2} (1+\xi\xi_k)(1-\eta^2) + \frac{\eta_k^2}{2} (1+\eta\eta_k)(1-\xi^2)$$
(5-5)

where  $\xi_k$ ,  $\eta_k$  represent the curvilinear coordinates of one of the eight boundary nodes, denoted by node *k*. The hierarchic shape function for the central ninth node is given by the following bubble function:

$$N_{9}(\xi,\eta) = (1-\xi^{2})(1-\eta^{2})$$
(5-6)

Note that in the presented shell element formulation, variables associated with the central ninth node represent relative displacements and do not coincide with interpolated Serendipity

displacement values. To calculate the actual displacement vector (displacements and rotations) associated with the ninth node, Equation 5-7 must be applied:

$$\{\delta_9\} = \sum_{k=1}^8 N_k(0,0)\{\delta_k\} + \{\Delta\delta_9\}$$
(5-7)

where,

 $\{\delta_9\}$  = the vector of actual displacements corresponding to the central ninth node;  $N_k$  = the Serendipity shape functions pertaining to boundary nodes (Equations 5-4 and 5-5);  $\{\delta_k\}$  = the vector of displacements corresponding to boundary node *k*;  $\{\Delta\delta_9\}$  = the 'displacements' associated with the central ninth node.

The geometry of all three degenerated shells is interpolated using only the eight Serendipity shape functions. However, the Heterosis element displacement field is developed through constraint of the translational displacements of the central ninth node ( $\Delta u_9 = \Delta v_9 = \Delta w_9 = 0$ ).

### 5.1.4 Coordinate Systems

Four coordinate systems are used to describe the shell element geometry and orientation in space: (i) a global Cartesian coordinate set, (ii) a nodal coordinate system, (iii) a curvilinear coordinate system, and (iv) a local coordinate system. The application of each coordinate system as described by Figueiras and Owen (1984) is briefly summarized below (see Figure 5.5).

### (i) Global Coordinate System (x, y, z)

The global coordinate system is a Cartesian coordinate set chosen freely during model generation to specify element geometry and orientation. The nodal coordinates and displacements, the global stiffness matrix, and the applied force vector refer to this system.

### (ii) Nodal Coordinate System (v_{1k}, v_{2k}, v_{3k})

A unique nodal coordinate system is defined at each nodal point (see Figure 5.5b), with the origin of the system located at the element's midsurface. The vector identified as  $\{v_{3k}\}$  is oriented in the out-of-plane (through thickness) direction, and is positive facing toward the top surface of the element, at node k. As such:





$$\{v_{3k}\} = \{x_{ik}\}^{top} - \{x_{ik}\}^{bot}$$
(5-8a)

where  $\{x_{ik}\}^{top}$  and  $\{x_{ik}\}^{bot}$  are the vectors of nodal coordinates of the top and bottom surfaces at node k, defined by Equation 5-8b.

$$\left\{x_{ik}\right\}^{top,bot} = \begin{cases}x_k\\y_k\\z_k\end{cases}^{top,bot}$$
(5-8b)

Vector  $\{v_{1k}\}$  is defined perpendicular to vector  $\{v_{3k}\}$ , and parallel to the global *x-z* plane.

$$v_{1k}^{x} = v_{3k}^{z}$$
;  $v_{1k}^{y} = 0$ ;  $v_{1k}^{z} = -v_{3k}^{x}$  (5-9)

where the superscripts refer to the vector components in the global coordinate system. In the case where  $\{v_{3k}\}$  is oriented in the global 'y' direction, then  $\{v_{1k}\}$  is oriented in the global 'x' direction:

$$v_{1k}^x = -v_{3k}^y$$
;  $v_{1k}^y = v_{1k}^z = 0$  (5-10)

Lastly, the remaining vector,  $\{v_{2k}\}$ , is defined perpendicular to the plane formed by vectors  $\{v_{1k}\}$ and  $\{v_{3k}\}$ :

$$\{v_{2k}\} = \{v_{3k}\} \times \{v_{1k}\}$$
(5-11)

The resulting vectors  $\{v_{1k}\}$ ,  $\{v_{2k}\}$ , and  $\{v_{3k}\}$  are illustrated in Figure 5.5b. It is important to note that vector  $\{v_{3k}\}$ , which is defined in the out-of-plane direction, represents the 'normal' at node k, and is not necessarily perpendicular to the midsurface at the location of node k. An advantage of the methodology used in defining  $\{v_{3k}\}$  is that resulting meshes consisting of multiple shell elements will possess no gaps and no overlap between adjacent elements. Vectors  $\{v_{1k}\}$  and  $\{v_{2k}\}$  also define the rotations  $\theta_{2k}$  and  $\theta_{1k}$ , respectively, where  $\theta_{2k}$  is the rotation about  $\{v_{1k}\}$ , and  $\theta_{1k}$  is the rotation about  $\{v_{2k}\}$ .

### (iii) Curvilinear Coordinate System $(\xi, \eta, \zeta)$

The normalized curvilinear coordinate system is illustrated in Figure 5.5b and Figure 5.6. In this system the midsurface plane of the shell is represented by variables  $\xi$  and  $\eta$ , and  $\zeta$  represents the linear coordinates in the thickness direction. With the assumption that  $\xi$ ,  $\eta$ , and  $\zeta$  vary from -1 to +1 in their respective directions, a consistent definition for element shape functions, element geometries, and stress resultants can be considered for all shells comprising a structure. Note that the  $\zeta$  - direction should be considered only approximately perpendicular to the elements midsurface since  $\zeta$  is defined as a function of  $\{v_{3k}\}$  (see Section 5.1.4(ii)).

# (iv) Local Coordinate System (x', y', z')

The local coordinate system is defined within each layer (to be discussed in Section 5.1.5), and at each of the nine Gauss point sampling locations where stresses and strains are calculated.



Figure 5.6 – Curvilinear Coordinates (adapted from Polak and Vecchio, 1993a)

The z' direction is taken perpendicular to the planar surface of the shell ( $\zeta$  = constant surface) (see Figure 5.6), and is calculated from the cross-product of the  $\xi$  and  $\eta$  directions:

$$\{z'\} = \begin{vmatrix} \frac{\partial x}{\partial \xi} \\ \frac{\partial y}{\partial \xi} \\ \frac{\partial z}{\partial \xi} \end{vmatrix} \times \begin{vmatrix} \frac{\partial x}{\partial \eta} \\ \frac{\partial y}{\partial \eta} \\ \frac{\partial z}{\partial \eta} \end{vmatrix}$$
(5-12)

The evaluation of the local direction  $\{x'\}$  is performed in the same manner as that which was used to define vector  $\{v_{1k}\}$ . Vector  $\{x'\}$  is defined perpendicular to vector  $\{z'\}$ , and parallel to the global *x-z* plane. In the case where  $\{z'\}$  is oriented in the global 'y' direction, then  $\{x'\}$  is oriented in the global 'x' direction. In the same manner as in the evaluation of the curvilinear system, vector  $\{y'\}$  is defined as the cross-product of  $\{x'\}$  and  $\{z'\}$ :

$$\{y'\} = \{z'\} \times \{x'\}$$
(5-13)

#### 5.1.5 Displacement Field

The displacement field of the shell element is described by five dof at each node: three translations (u, v, w) and two rotations  $(\theta_1, \theta_2)$ . Displacement contributions occurring over the thickness of the element (or along the 'normal') as a result of element rotations are defined by the following, on the basis of a 'small rotations' assumption:

$$\delta_{1k} = h\theta_{1k} \quad ; \quad \delta_{2k} = h\theta_{2k} \tag{5-14}$$

where,

 $\delta_{1k}$  = the rotational displacement contribution in the direction of vector { $v_{1k}$ };  $\delta_{2k}$  = the rotational displacement contribution in the direction of vector { $v_{2k}$ };

*h* = the distance from the midsurface of the element (i.e., from surface  $\zeta = 0$ ).

The rotational contributions to the displacements in the global system are calculated using Equation 5-15.

$$\{\delta_i\}_{\theta_{1k}} = \delta_{1k} v_{1k}^i \quad ; \quad \{\delta_i\}_{\theta_{2k}} = \delta_{2k} (-v_{2k}^i) \tag{5-15}$$

where  $v_{1k}^i$  and  $v_{2k}^i$  represent the *i*th coordinates of unit vectors in the directions of  $\{v_{1k}\}$  and  $\{v_{2k}\}$ . Note that i = x, y, z from the global reference system. The element displacement field can then be expressed by:

$$\begin{cases} u \\ v \\ w \end{cases} = \sum_{k=1}^{n} N_{k} \begin{cases} u_{k} \\ v_{k} \\ w_{k} \end{cases} + \sum_{k=1}^{n} N_{k} \zeta \frac{h_{k}}{2} \begin{bmatrix} v_{1k}^{x} & v_{2k}^{x} \\ v_{1k}^{y} & v_{2k}^{y} \\ v_{1k}^{z} & v_{2k}^{z} \end{bmatrix} \begin{cases} \theta_{1k} \\ -\theta_{2k} \end{cases}$$
(5-16)

The contribution to the global displacements from a single node k can be expressed as follows:

$$\begin{cases} u \\ v \\ w \end{cases} = \begin{bmatrix} N_k & 0 & 0 & N_k \zeta \frac{h_k}{2} v_{1k}^x & -N_k \zeta \frac{h_k}{2} v_{2k}^z \\ 0 & N_k & 0 & N_k \zeta \frac{h_k}{2} v_{1k}^y & -N_k \zeta \frac{h_k}{2} v_{2k}^y \\ 0 & 0 & N_k & N_k \zeta \frac{h_k}{2} v_{1k}^z & -N_k \zeta \frac{h_k}{2} v_{2k}^z \end{bmatrix} \begin{bmatrix} u_k \\ v_k \\ w_k \\ \theta_{1k} \\ \theta_{2k} \end{bmatrix}$$
(5-17a)

where  $h_k$  is the total thickness of element at the location of node k. Expressed more generally:

$$\left\{u_{k}\right\} = \left[N_{k}\right]\left\{\delta_{k}\right\} \tag{5-17b}$$

In similar fashion, the displacements for the complete shell element can be expressed as:

$$\{u\} = [N]\{\delta\} \tag{5-18}$$

### 5.1.6 Stresses and Strains

To ease the treatment of the assumption that stresses normal to the out-of-plane direction (i.e., in the z' direction) are negligible, element stresses and strains are defined in the local x', y', z' coordinate system. The five local strain components are given by Equation 5-19, where u', v', w' represent the local displacement components. Note that shear strains are expressed using engineering notation throughout the thesis.

$$\left\{\varepsilon\right\} = \begin{cases} \varepsilon_{x'} \\ \varepsilon_{y'} \\ \gamma_{x'y'} \\ \gamma_{x'z'} \\ \gamma_{y'z'} \end{cases} = \begin{cases} \frac{\partial u'}{\partial x'} \\ \frac{\partial v'}{\partial y'} \\ \frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'} \\ \frac{\partial u'}{\partial z'} + \frac{\partial w'}{\partial x'} \\ \frac{\partial v'}{\partial z'} + \frac{\partial w'}{\partial x'} \\ \frac{\partial v'}{\partial z'} + \frac{\partial w'}{\partial y'} \end{cases}$$
(5-19)

With reference to the curvilinear coordinate system, the global displacement derivatives can be determined from Equation 5-16. With the use of an appropriate Jacobian matrix, [J], displacement derivatives relative to the global coordinate set can be evaluated:

$$\begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} & \frac{\partial w}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} & \frac{\partial w}{\partial y} \\ \frac{\partial u}{\partial z} & \frac{\partial v}{\partial z} & \frac{\partial w}{\partial z} \end{bmatrix} = \begin{bmatrix} J^{-1} \end{bmatrix} \begin{bmatrix} \frac{\partial u}{\partial \xi} & \frac{\partial v}{\partial \xi} & \frac{\partial w}{\partial \xi} \\ \frac{\partial u}{\partial \eta} & \frac{\partial v}{\partial \eta} & \frac{\partial w}{\partial \eta} \\ \frac{\partial u}{\partial \zeta} & \frac{\partial v}{\partial \zeta} & \frac{\partial w}{\partial \zeta} \end{bmatrix} ; \qquad \begin{bmatrix} J \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial \xi} & \frac{\partial v}{\partial \xi} & \frac{\partial w}{\partial \xi} \\ \frac{\partial u}{\partial \eta} & \frac{\partial v}{\partial \eta} & \frac{\partial w}{\partial \eta} \\ \frac{\partial u}{\partial \zeta} & \frac{\partial v}{\partial \zeta} & \frac{\partial w}{\partial \zeta} \end{bmatrix}$$
(5-20)

The local displacement derivatives required to calculate the local strains (Equation 5-19), can be calculated from transformation from the global values determined above in Equation 5-20:

$$\begin{bmatrix} \frac{\partial u'}{\partial x'} & \frac{\partial v'}{\partial x'} & \frac{\partial w'}{\partial x'} \\ \frac{\partial u'}{\partial y'} & \frac{\partial v'}{\partial y'} & \frac{\partial w'}{\partial y'} \\ \frac{\partial u'}{\partial z'} & \frac{\partial v'}{\partial z'} & \frac{\partial w'}{\partial z'} \end{bmatrix} = \begin{bmatrix} T \end{bmatrix}^T \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} & \frac{\partial w}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} & \frac{\partial w}{\partial y} \\ \frac{\partial u}{\partial z} & \frac{\partial v}{\partial z} & \frac{\partial w}{\partial z} \end{bmatrix} \begin{bmatrix} T \end{bmatrix}$$
(5-21)

where [T] is the transformation matrix relating the local x', y', z' coordinate system to the global x, y, z system. The strain-displacement matrix, [B], is formulated such that local strains are evaluated directly from nodal global displacement values, and as such:

$$\{\varepsilon\} = [B]\{\delta\}$$
(5-22)

Lastly, the element stresses are also calculated with respect to the local axis system to ensure that the zero out-of-plane stress ( $\sigma_z$ ' = 0) condition is enforced. The five local stresses are presented in Equation 5-23.

$$\{\sigma\} = \begin{cases} \sigma_{x'} \\ \sigma_{y'} \\ \tau_{x'y'} \\ \tau_{x'z'} \\ \tau_{y'z'} \end{cases} = [D]\{\varepsilon\}$$

$$(5-23)$$

In the above equation, the local strain matrix,  $\{\varepsilon\}$ , is the same as that presented in Equation 5-19. The material matrix, [D], is formulated in accordance with the governing material behavioural and constitutive models. The development of [D] will be discussed in Section 5.3.

#### 5.1.7 Layered Element Approach

To account for non-uniform variations of stresses and material stiffnesses through the thickness of the shell, a layered approach is adopted. Local stresses are assumed to be constant over the height of an individual layer and, as such, are integrated using a single sampling point located at the midheight of each layer (see Figure 5.7). In-plane reinforcement can be defined in any planar orientation and is incorporated discretely within the thickness of the element (i.e., in-plane reinforcement is not smeared throughout concrete layers). Transverse reinforcement oriented in the out-of-plane direction (z' direction) is treated in a smeared sense, and is specified as a property of the concrete layers. The number of layers required to effectively capture the nonlinear stress variation and variation in material stiffness is problem dependent.

Stresses, strains, and the material stiffness matrices are calculated at the midpoint of each concrete and steel layer. The total global stiffness matrix pertaining to an individual Gauss point (see Section 5.1.3) is developed through layer-wise integration of the material stiffnesses. Stress resultants pertaining to any individual Gauss sampling point are calculated in the same manner (adapted from Figueiras and Owen, 1984):



(a) layers through thickness(b) concrete stresses(c) steel stressesFigure 5.7 – Layered Modelling (adapted from Polak and Vecchio, 1993a)

normal  
forces: 
$$N_{x(y)} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{x(y)} dz = \frac{h}{2} \sum_{i=1}^{n} \sigma_{x(y)}^{i} \Delta \zeta^{i}$$
(5-24a)

in-plane shear:

$$N_{xy} = N_{yx} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xy} dz = \frac{h}{2} \sum_{i=1}^{n} \tau_{xy}^{i} \Delta \zeta^{i}$$
(5-24b)

*bending moments:* 

$$M_{x(y)} = -\int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{x(y)} z dz = -\frac{h^2}{4} \sum_{i=1}^{n} \sigma_{x(y)}^i \zeta^i \Delta \zeta^i$$
(5-24c)

twisting moments:

$$M_{xy} = M_{yx} = -\int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xy} z dz = -\frac{h^2}{4} \sum_{i=1}^{n} \tau_{xy}^i \zeta^i \Delta \zeta^i$$
(5-24d)

out-of-plane 
$$V_{xz(yz)} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xz(yz)} dz = \frac{h}{2} \sum_{i=1}^{n} \tau_{xz(yz)}^{i} \Delta \zeta^{i}$$
(5-24e)  
shears:

where h is equal to the total section thickness measured in the out-of-plane (z') direction. In Equations 5-24a through e, the membrane forces have been separated into normal and in-plane shear forces. The moments acting on the section have been separated into bending and twisting moments. The sign convention for the stress resultants presented above is illustrated in Figure 5.8.

### 5.1.8 Geometric Nonlinearity

The inclusion of a nonlinear geometry formulation in the analysis of shell and plate structures can significantly impact the accuracy of the analytical results. Given that shell elements are often subjected to combined in-plane and out-of-plane stresses, geometric instability arising from large

deformations is a relevant failure condition. The current method used to incorporate geometric nonlinearity in VecTor4 considers a total Lagrangian formulation, and is the same as that initially implemented by Figueiras and Owen (1984). As such, this section is intended to provide only a brief summary of the method.



Figure 5.8 – Stress Resultant Sign Convention (adapted from Figueiras and Owen, 1984)

The total Lagrangian formulation refers stresses and strains to the original geometry of the system, but constantly updates the displacement field with respect to the current geometry. The strain-displacement matrix consists of two parts: a linear portion which is evaluated once at the beginning of the analysis using the initial geometry of the structure, and a nonlinear portion which is updated over the course of the analysis.

For convenience, the system strains are expressed using Green-Lagrange strain notation in which the linear and nonlinear terms are separated:

$$\{\varepsilon\} = \left\langle \varepsilon_{x'} \quad \varepsilon_{y'} \quad \gamma_{x'y'} \quad \gamma_{x'z'} \quad \gamma_{y'z'} \right\rangle$$

$$= \left\langle \frac{\partial u'}{\partial x'} \quad \frac{\partial v'}{\partial y'} \quad \frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial x'} \quad \frac{\partial u'}{\partial z'} + \frac{\partial w'}{\partial x'} \quad \frac{\partial v'}{\partial z'} + \frac{\partial w'}{\partial y'} \right\rangle + \left\langle \frac{1}{2} \left( \frac{\partial w'}{\partial x'} \right)^2 \quad \frac{1}{2} \left( \frac{\partial w'}{\partial y'} \right)^2 \quad \frac{\partial w'}{\partial x'} \frac{\partial w'}{\partial y'} \quad 0 \quad 0 \right\rangle$$

$$\{\varepsilon\} = \{\varepsilon_l\} + \{\varepsilon_{nl}\}$$

$$(5-25b)$$

Consequently, with reference to Equation 5-25, the strain-displacement matrix takes the following form:

$$\begin{bmatrix} B \end{bmatrix} = \begin{bmatrix} B_l \end{bmatrix} + \begin{bmatrix} B_{nl} \end{bmatrix}$$
(5-26)

where  $[B_l]$  represents the linear portion of the matrix and  $[B_{nl}]$  represents the nonlinear portion of the matrix. The variation of the  $\{\varepsilon\}$  yields Equation 5-27, which in turn is used to develop the virtual work representation of the system presented in Equation 5-28.

$$\{\delta\varepsilon\} = [B]\{\delta d\} + [\delta B]\{d\}$$
(5-27)

$$\int_{v} \langle \delta \varepsilon \rangle [D] \{\varepsilon\} dv = \langle \delta d \rangle \int_{v} [B]^{T} [D] [B] dv \{d\} + \langle d \rangle \int_{v} [\delta B]^{T} \{\sigma\} dv$$
(5-28)

The first part of the right-hand side of Equation 5-28 is the typical work expression for the elemental stiffness matrix,  $[k_o]$ . The second part of the equation represents the stress dependent, geometric portion of the elemental stiffness matrix,  $[k_\sigma]$ . Therefore, the complete elemental stiffness matrix which includes consideration of geometric nonlinearity is calculated from:

$$[k] = [k_o] + [k_\sigma]$$
(5-29)

Since the nonlinear portion of the stiffness matrix is dependent on the current state of stress, the nonlinear portion of the strain-displacement matrix must be updated in each iteration. This is of little concern however, as the material matrix required to calculate  $[k_o]$  also requires updating in each iteration to account for material nonlinearity. The consideration of material nonlinearity will be discussed in the following sections.

## **5.2 Governing Behavioural Models**

As outlined is Section 5.1, the specific nature of the shell finite element forming the analytical software program can impact the program's ability to accurately model the behaviour of shell structures. However, perhaps the most significant aspect in the analytical modelling of RC shells using VecTor4 is attributed to the governing behavioural and constitutive models considered in the program. In VecTor4, the behaviour of reinforced concrete is based principally on the formulations of the Modified Compression Field Theory (MCFT) and the Disturbed Stress Field Model (DSFM).

This section provides a limited presentation of the MCFT and DSFM formulations with the primary emphasis being attributed to highlighting the differences amongst the two models. Both models have been heavily cited, referenced, and discussed throughout the database of reinforced concrete literature. As such, detailed formulations pertaining to the two models are not been provided in this thesis.

# 5.2.1 The Modified Compression Field Theory (MCFT)

The MCFT, originally published in the 1980s by Vecchio and Collins (1986), serves as a generalized approach for modelling the behaviour of reinforced concrete elements subjected to biaxial loading conditions. Central to the theory is the treatment of cracked reinforced concrete as a new orthotropic material with its own set of governing constitutive equations. The application of a smeared, rotating crack model treats stresses and strains in an average sense, and allows stresses and strains to gradually reorient as a result of changing load and/or material response. The composite element, which consists of cracked concrete and in-plane reinforcement, is governed by equations of equilibrium, compatibility, and constitutive material models. Though cracks are considered smeared, and stresses and strains are averaged, a key feature of the model is the examination of local behaviour at crack locations.

The governing constitutive models of the MCFT were developed empirically from the original series of reinforced concrete panel elements tested at the University of Toronto (Vecchio and Collins, 1986). Since then, the formulations have been corroborated by more than 200 tests of reinforced concrete panel elements (see Figure 5.9).



Figure 5.9 – Panel Element Tester (University of Toronto)

Key assumptions in the MCFT include:

- reinforcement is distributed uniformly throughout the element;
- cracks are smeared across the element and free to rotate;
- stresses (in-plane normal and shear) are applied uniformly;
- a unique strain state exists for each stress state; strain history is not considered;
- stresses and strains are averaged over a distance that includes several cracks;
- perfect bond conditions exist between concrete and reinforcement;
- reinforcement develops axial stresses only (shear stresses ignored);
- axes of principal stress and principal strain are coincident;
- constitutive relationships for concrete and reinforcement are independent.

As noted above, the original series of panel tests performed by Vecchio and Collins led to the development of new constitutive formulations. The constitutive models, which were developed from tests of cracked reinforced concrete elements under biaxial loading, were markedly different from relationships commonly used to describe the behaviour of elements under uniaxial loading conditions.

When subjected to large tensile strains normal to the direction of principal compression, cracked concrete exhibited a softer and much weaker compressive response in comparison to that of uniaxially loaded concrete (see Figure 5.10a). This phenomenon led to the development of a

compression softening relationship which was incorporated into the constitutive compressive response for cracked reinforced concrete.

With regard to the behaviour of cracked concrete subject to tension, it was found that even at large tensile strain values, appreciable tensile stresses were being carried by the concrete between crack locations. This observation led to the development of the tension stiffening formulation which represented the average tensile stress developed by cracked concrete as a result of tensile straining. The developed stress versus strain relationship for cracked concrete in tension is presented in Figure 5.10b.



(a) tension stiffening

Figure 5.10 – Stress-Strain Relationships for Cracked Reinforced Concrete (Vecchio and Collins, 1986)

Because compatibility, equilibrium, and constitutive formulations are developed on the basis of average stresses and average strains, the MCFT requires that examination of local crack conditions be performed to verify that the computed average stresses are compatible with the condition of the cracked concrete element.

In Figure 5.11a the average stresses in the concrete oriented perpendicular to the principal tensile strain are shown, and in Figure 5.11b the local stresses occurring on the crack surface are shown. As it is assumed that there is zero concrete tensile stress at the location of the crack, the average tensile stress,  $f_{c1}$ , must be carried by way of increased reinforcement stresses at the crack location. If the reinforcement reserve capacity is not sufficient to transfer the average tensile stress across the crack, then  $f_{c1}$  is reduced, ensuring equilibrium is preserved at the crack.



Figure 5.11 – Stresses of a Reinforced Concrete Element (adapted from Vecchio, 2000)

As a result of local stress increases in the reinforcement, shearing stress on the crack surface (denoted as  $v_{ci}$  in Figure 5.11b) may occur. If the amount of shear stress on the crack becomes sufficiently large, slip along the crack surface may develop. As a means of incorporating the effects of the local shear on the crack surface, the MCFT limits the allowable shear stress on the crack surface based on an aggregate interlock resistance formulation developed from experimental data provided by Walraven (1981). If the prescribed limit is exceeded, the value of  $f_{c1}$  is reduced until the shear on the crack surface satisfies the developed limit.

A concise presentation of the MCFT equations of element equilibrium, compatibility, and those pertaining to the developed constitutive models for cracked concrete and steel reinforcement is provided in Figure 5.12. Note that the crack spacing parameters  $s_x$  and  $s_y$  are typically calculated in accordance with the CEB-FIP formulations which are available in literature (Collins and Mitchell, 1997). Additionally, the equations provided in the figure differ slightly from those originally presented by Vecchio and Collins; however, the original formulation of the MCFT remains essentially unchanged.



Figure 5.12 – Equations of the MCFT (adapted from Bentz et al., 2006)

### 5.2.2 The Disturbed Stress Field Model (DSFM)

Since its inception the MCFT has been applied in a wide range of reinforced concrete analysis applications spanning various structure types and loading configurations. Its implementation in various analytical tools has shown that in most cases it provides reasonable estimates of load capacities and failure modes, and can be used to identify critical factors influencing the performance of reinforced concrete structures (Vecchio et al., 2004). Although in most applications, the use of the MCFT has generally resulted in high accuracy and reliability, some deficiencies pertaining to specific situations have been noted (Vecchio, 2000). For panel elements with high amounts of reinforcement, the MCFT has exhibited the tendency to underestimate the shear strength and stiffness; for panels with low amounts of reinforcement, shear strength and stiffness.

It has been suggested that the reduced accuracy of the model in these situations may in part be due to the assumption that the axes of the principal stresses and principal strains remain coincident throughout loading. In the initial series of panel tests performed by Vecchio and Collins it was shown that some level of deviation existed between the inclination of the principal stresses and the inclination of the principal strains (see Figure 5.13); however, it was felt that treating the orientations of the principal stresses and strains as coincident was a reasonable simplification of the problem at hand (Vecchio and Collins, 1986). The DSFM is essentially an extension of the formulations of the MCFT, with the removal of the restriction that the orientation of the MCFT which was formulated on the basis of reinforcement components being orthogonally aligned in the planar x and y directions, the equilibrium relations of the DSFM are cast such that any number of reinforcement components, oriented in any planar direction, can be considered.



Figure 5.13 – Deviation of Stresses and Strains (Vecchio and Collins, 1986; Vecchio, 2000)

To accommodate the rotational lag of the stress field observed from the experimental data, the DSFM incorporates deformations due to crack slip in the element compatibility relationships. As an added benefit of the explicit inclusion of the slip deformation on the crack surface, the crack slip check required in the MCFT need not be performed.

In the DSFM it is assumed that the strains acting on the element can be decomposed into two sets: total strains ( $\varepsilon_x$ ,  $\varepsilon_y$ ,  $\gamma_{xy}$ ) which represent measured deformations of the element, and net concrete strains ( $\varepsilon_{cx}$ ,  $\varepsilon_{cy}$ ,  $\gamma_{cxy}$ ) which represent the strain values to be used in conjunction with the constitutive models. The difference between the total and the net strains is effectively the smeared slip strain contribution resulting from concrete crack slip. The DSFM compatibility relationships governing a reinforced concrete membrane element are illustrated in Figure 5.14.



Figure 5.14 – DSFM Compatibility Relations (adapted from Vecchio, 2000)

Crack slip strains (denoted as  $\varepsilon_x^s$ ,  $\varepsilon_y^s$ ,  $\gamma_{xy}^s$  in Figure 5.14b) are calculated in an average (smeared) sense from the local slip displacements,  $\delta_s$ , occurring at the crack location. Adaptation of the aggregate interlock formulation considered in the MCFT, forms the basis of the local slip displacement calculation.

With the formulation of the DSFM, some modifications of the governing constitutive models and solution algorithms were required; mainly to accommodate the explicit inclusion of element slip deformations which are not considered in the formulations of the MCFT (Vecchio, 2001).

A comprehensive summary of the constitutive relations which are common to the suite of VecTor software programs (e.g., VecTor2, VecTor3, VecTor4, VecTor5, and VecTor6) are presented in Wong et al. (2012). However, in applying the constitutive models to analyze reinforced concrete elements under three-dimensional loading conditions, the relations pertaining to compression softening have been modified in VecTor4.

In the two-dimensional VecTor analysis programs the compression softening coefficient,  $\beta_d$ , for an element subjected to biaxial stress conditions is computed from (Wong et al., 2012):

$$\beta_d = \frac{1}{1 + C_s C_d} \le 1.0 \tag{5-30a}$$

$$C_d = 0.35 \left( -\varepsilon_{c1} / \varepsilon_{c2} - 0.28 \right)^{0.80}$$
(5-30b)

where  $\varepsilon_{c1}$  and  $\varepsilon_{c2}$  are the net tensile and net compressive strains in the concrete, respectively, and  $C_s$  is a factor which is used to recognize whether or not slip deformations are considered in the analysis (i.e., whether computations are in accordance with the MCFT ( $C_s = 1.00$ ) or in accordance with the DSFM ( $C_s = 0.55$ )).

To account for the presence of three-dimensional stress conditions in VecTor4, factor  $C_d$  presented in Equation 5-30b has been modified:

if 
$$\varepsilon_{c2} > 0$$
:  $C_d = 0.35 \left( -\sqrt{\varepsilon_{c1}^2 + \varepsilon_{c2}^2} / \varepsilon_{c3} - 0.28 \right)^{0.80}$  (5-31a)

if 
$$\varepsilon_{c2} \le 0$$
:  $C_d = 0.35 \left(-\varepsilon_{c1}/\varepsilon_{c3} - 0.28\right)^{0.80}$  (5-31b)

Note that if the intermediate principal concrete strain value,  $\varepsilon_{c2}$ , is compressive or zero, Equation 5-31 is of the same form as that for an element under biaxial stress conditions. If  $\varepsilon_{c1}$  and  $\varepsilon_{c2}$  are

both tensile, the compression softening factor is a function of an effective tensile strain which is computed as the vector sum of principal tensions. This effective tensile strain approach has been used previously to analyze reinforced concrete elements under three-dimensional loading conditions in accordance with the MCFT (Kirschner and Collins, 1986; Adebar and Collins, 1991; Polak and Vecchio, 1993b).

### **5.3 VecTor4 Finite Element Implementation**

This section presents an overview of the solution algorithm adopted in VecTor4. The methodology used to develop the material matrix, [D], is primarily based on that developed by Vecchio (2001) for the DSFM; however, in VecTor4, analyses considering the DSFM or the MCFT are accommodated using the presented solution procedure.

## **5.3.1 Material Matrix Development**

In VecTor4 each concrete and steel layer is analyzed individually and, as such, comprises a unique contribution toward the overall stiffness matrix forming the shell element. Material matrices for concrete and steel layers are developed separately; however, the material behavioural responses of the layers are somewhat interdependent.

For the purpose of evaluating the behaviour of the concrete, steel reinforcement layers are treated in a smeared sense over the thickness and across the width of the element. In the direction normal to the plane of the element, the effective tributary area pertaining to an individual steel component is assumed to span 7.5 times the bar diameter in each through-thickness direction, in accordance with CEB-FIP (1990) modelling guidelines (see Figure 5.15). For simplicity, reinforcement is uniformly smeared across the planar width of the element. Concrete layers that fall within the tributary reinforcement area consider tension stiffening effects and require examination of the local behaviour at the crack location. It is important to note that the actual stiffness contribution from the steel layers is not smeared through the thickness of the element, but is treated as its own discrete layer within the RC element.



Figure 5.15 – Effective Smeared Reinforcement Regions

In the general case, the total strains,  $\{\varepsilon\}$ , acting within an arbitrary concrete layer of the shell element can be represented as a composition of: i) concrete net strains,  $\{\varepsilon_c\}$ , ii) concrete elastic offsets,  $\{\varepsilon_c^o\}$  (thermal effects, shrinkage strains, lateral expansions such as Poisson's effect, and prestrains), iii) concrete plastic offsets,  $\{\varepsilon_c^p\}$  (permanent damage resulting from strain history), and iv) concrete crack slip offset strains which result from shear slip on the crack surface,  $\{\varepsilon_c^s\}$ (considered in the formulations of the DSFM). The total concrete strains can be expressed as:

$$\left\{\varepsilon\right\} = \left\{\varepsilon_{c}\right\} + \left\{\varepsilon_{c}^{o}\right\} + \left\{\varepsilon_{c}^{p}\right\} + \left\{\varepsilon_{c}^{s}\right\} = \left\langle\varepsilon_{x} \quad \varepsilon_{y} \quad \varepsilon_{z} \quad \gamma_{xy} \quad \gamma_{xz} \quad \gamma_{yz}\right\rangle$$
(5-32)

Assuming that reinforcement and concrete possess perfect bond, for a given reinforced concrete layer, the total strains developed in the *i*th layer of reinforcement are equal to the total strain of the concrete at the same location. In a manner similar to that employed for the concrete strains, the total strain in the reinforcement can be expressed as: i) reinforcement net strains,  $\{\varepsilon_s\}_i$ , ii) reinforcement elastic offsets,  $\{\varepsilon_s^o\}_i$  (due to thermal effects and prestrains), and iii) reinforcement plastic offsets,  $\{\varepsilon_s^p\}_i$ , (resulting from steel yielding or damage resulting from strain history):

$$\left\{\varepsilon\right\}_{i} = \left\{\varepsilon_{s}\right\}_{i} + \left\{\varepsilon_{s}^{o}\right\}_{i} + \left\{\varepsilon_{s}^{p}\right\}_{i} = \left\langle\varepsilon_{x} \quad \varepsilon_{y} \quad \varepsilon_{z} \quad \gamma_{xy} \quad \gamma_{xz} \quad \gamma_{yz}\right\rangle_{i}$$
(5-33)

Note that, as defined in Equation 5-19, the local out-of-plane strain term,  $\varepsilon_z$ , is not calculated directly from the displacement field of the shell element. The value of  $\varepsilon_z$  as shown in Equation 5-32 is determined from a previous iteration of the solution algorithm, or the final iteration of a

previous load stage, on the basis of the zero out-of-plane stress assumption. The computation method used to evaluate  $\varepsilon_z$  is provided in Section 5.3.2.

From the strain condition presented in the Equation 5-32, the concrete principal strains  $(\varepsilon_{c1}, \varepsilon_{c2}, \varepsilon_{c3})$  and their corresponding direction cosine vectors are computed. Principal stresses  $(f_{c1}, f_{c2}, f_{c3})$  are computed in accordance with the formulations of the MCFT or DSFM behavioural models presented in Section 5.2. It is important to note that the following convention has been adopted for the identification of the principal stresses and strains:

$$\begin{cases} \varepsilon_{c1} \\ \varepsilon_{c2} \\ \varepsilon_{c3} \end{cases} = \begin{cases} maximum principal strain \\ intermediate principal strain \\ minimum principal strain \end{cases} ; \begin{cases} f_{c1} \\ f_{c2} \\ f_{c3} \end{cases} = \begin{cases} stress corresponding to \varepsilon_{c1} \\ stress corresponding to \varepsilon_{c2} \\ stress corresponding to \varepsilon_{c3} \end{cases}$$
(5-34)

Secant moduli pertaining to the concrete material stiffnesses in the principal stress directions are calculated in accordance with Figure 5.16a, using the following:

$$\overline{E}_{c1} = \frac{f_{c1}}{\varepsilon_{c1}} \quad ; \quad \overline{E}_{c2} = \frac{f_{c2}}{\varepsilon_{c2}} \quad ; \quad \overline{E}_{c3} = \frac{f_{c3}}{\varepsilon_{c3}} \tag{5-35}$$

The secant shear moduli are calculated according to:

$$\overline{G}_{c12} = \frac{\overline{E}_{c1} \cdot \overline{E}_{c2}}{\overline{E}_{c1} + \overline{E}_{c2}} \quad ; \quad \overline{G}_{c13} = \frac{\overline{E}_{c1} \cdot \overline{E}_{c3}}{\overline{E}_{c1} + \overline{E}_{c3}} \quad ; \quad \overline{G}_{c23} = \frac{\overline{E}_{c2} \cdot \overline{E}_{c3}}{\overline{E}_{c2} + \overline{E}_{c3}} \tag{5-36}$$



Figure 5.16 – Defining Secant Moduli (a) concrete (b) reinforcement (adapted from Wong et al., 2012)

As the MCFT and DSFM consider reinforced concrete as an orthotropic material in the principal stress directions, the concrete material matrix,  $[D_c]$ ', is formulated with respect to the orientation of principal stress, as follows:

$$\begin{bmatrix} D_c \end{bmatrix}' = \begin{bmatrix} \overline{E}_{c1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \overline{E}_{c2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{E}_{c3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{G}_{c12} & 0 & 0 \\ 0 & 0 & 0 & 0 & \overline{G}_{c13} & 0 \\ 0 & 0 & 0 & 0 & 0 & \overline{G}_{c23} \end{bmatrix}$$
(5-37)

Recall that in this formulation, Poisson's effect is treated as an elastic offset and, as such, the material matrix presented in Equation 5-37 will always be diagonal. Transformation to the local coordinate system (x', y', z') (as defined in Section 5.1.4 (ii)) is performed using the transformation matrix,  $[T_c]$ :

$$\begin{bmatrix} D_c \end{bmatrix} = \begin{bmatrix} T_c \end{bmatrix}^T \begin{bmatrix} D_c \end{bmatrix}' \begin{bmatrix} T_c \end{bmatrix}$$
(5-38)

where the matrix  $[T_c]$  is developed using direction cosines  $(l_i, m_i, n_i)$  relating the axes of principal stress to the local x', y', z' system.  $[T_c]$  is constructed as follows (Cook et al., 1989):

$$[T_{c}] = \begin{bmatrix} l_{1}l_{1} & m_{1}m_{1} & n_{1}n_{1} & l_{1}m_{1} & l_{1}n_{1} & m_{1}n_{1} \\ l_{2}l_{2} & m_{2}m_{2} & n_{2}n_{2} & l_{2}m_{2} & l_{2}n_{2} & m_{2}n_{2} \\ l_{3}l_{3} & m_{3}m_{3} & n_{3}n_{3} & l_{3}m_{3} & l_{3}n_{3} & m_{3}n_{3} \\ 2l_{1}l_{2} & 2m_{1}m_{2} & 2n_{1}n_{2} & (l_{1}m_{2}+l_{2}m_{1}) & (l_{1}n_{2}+l_{2}n_{1}) & (m_{1}n_{2}+m_{2}n_{1}) \\ 2l_{1}l_{3} & 2m_{1}m_{3} & 2n_{1}n_{3} & (l_{3}m_{1}+l_{1}m_{3}) & (l_{3}n_{1}+l_{1}n_{3}) & (m_{3}n_{1}+m_{1}n_{3}) \\ 2l_{2}l_{3} & 2m_{2}m_{3} & 2n_{2}n_{3} & (l_{2}m_{3}+l_{3}m_{2}) & (l_{2}n_{3}+l_{3}n_{2}) & (m_{2}n_{3}+m_{3}n_{2}) \end{bmatrix}$$

$$(5-39)$$

The same approach used to develop the local concrete material matrix  $[D_c]$ , is employed for the in-plane reinforcement layers. In accordance with Figure 5.16b, the secant modulus for a single layer of in-plane reinforcement is defined as:

$$\overline{E}_s = \frac{f_s}{\varepsilon_s}$$
(5-40)

As it is assumed that reinforcing bars carry no appreciable shear stress (i.e., dowel stresses), the reinforcement material matrix can be developed with respect to the longitudinal axis of the reinforcement component considering only the secant modulus calculated in Equation 5-40:

where  $\rho_s$  is the reinforcement ratio corresponding to the layer of reinforcement under consideration. Using the same transformation concept applied for the concrete material matrix, the local material matrix developed for a layer of in-plane reinforcement is:

$$\begin{bmatrix} D_s \end{bmatrix} = \begin{bmatrix} T_s \end{bmatrix}^T \begin{bmatrix} D_s \end{bmatrix}' \begin{bmatrix} T_s \end{bmatrix}$$
(5-42)

Lastly, if out-of-plane reinforcement is present in the concrete layer under consideration, the local material matrix,  $[D_z]$ , can be determined directly without transformation since VecTor4 only considers out-of-plane reinforcement oriented in the local z' direction:

Note that  $\overline{E}_z$  is calculated in the same manner as the secant modulus for in-plane reinforcement contributions (see Equation 5-40 and Figure 5.16b), and  $\rho_z$  represents the out-of-plane reinforcement ratio.

In VecTor4, in-plane reinforcement components are treated discretely within the thickness of the RC elements by way of the discrete layered approach. As such, no benefit is achieved from the construction of the composite material matrix as is employed in the original publication of the DSFM (Vecchio, 2001). Material stresses are calculated separately, and material stiffness contributions from in-plane reinforcement components are incorporated separately from concrete in the construction of the global stiffness matrix. However, because the presence of out-of-plane reinforcement is treated as a smeared property of the concrete layers, a single material matrix,  $[D_{cz}]$ , is used to represent the combined material stiffness comprised of contributions from the concrete and the out-of-plane reinforcement:

$$\begin{bmatrix} D_{cz} \end{bmatrix} = \begin{bmatrix} D_c \end{bmatrix} + \begin{bmatrix} D_z \end{bmatrix}$$
(5-44)

The stresses acting within the concrete layers can be calculated using Equation 5-45.

$$\{\sigma_{cz}\} = [D_{cz}]\{\varepsilon\} - \{\sigma_{cz}^o\}$$
(5-45)

Because the displacement field of the shell element is developed from the consideration of total strains, both the  $[D_{cz}]$  matrix and  $\{\varepsilon\}$  vector in Equation 5-45 must also be formulated in terms of total strains. To accommodate the strain offsets presented in Equation 5-32, the pseudo-stress vector,  $\{\sigma_{cz}^o\}$ , is developed and included in the above equation. The pseudo-stresses pertaining to the concrete layers can be calculated as follows:

$$\left\{\sigma_{cz}^{o}\right\} = \left\{\sigma_{c}^{o}\right\} + \left\{\sigma_{z}^{o}\right\} = \left\langle\sigma_{cz_{x}}^{o} \quad \sigma_{cz_{y}}^{o} \quad \sigma_{cz_{z}}^{o} \quad \tau_{cz_{xy}}^{o} \quad \tau_{cz_{xz}}^{o} \quad \tau_{cz_{yz}}^{o}\right\rangle$$
(5-46a)

$$\left\{\sigma_{c}^{o}\right\} = \left[D_{c}\right]\left(\left\{\varepsilon_{c}^{o}\right\} + \left\{\varepsilon_{c}^{p}\right\} + \left\{\varepsilon_{c}^{s}\right\}\right)$$
(5-46b)

$$\left\{\sigma_{z}^{o}\right\} = \left[D_{z}\right]\left(\left\{\varepsilon_{z}^{o}\right\} + \left\{\varepsilon_{z}^{p}\right\}\right)$$
(5-46c)

Therefore, the stresses in the concrete layers representing the combined stiffness of the concrete and the out-of-plane reinforcement can be expressed as:

$$\begin{cases} \sigma_{cz_{x}} \\ \sigma_{cz_{y}} \\ \sigma_{cz_{x}} \\ \tau_{cz_{xy}} \\ \tau_{cz_{xz}} \\ \tau_{cz_{yz}} \end{cases} = \begin{bmatrix} D_{cz} \end{bmatrix} \begin{cases} \varepsilon_{x} \\ \varepsilon_{y} \\ \varepsilon_{z} \\ \gamma_{xy} \\ \gamma_{yz} \end{cases} - \begin{cases} \sigma_{cz_{x}}^{o} \\ \sigma_{cz_{y}}^{o} \\ \sigma_{cz_{z}}^{o} \\ \tau_{cz_{xy}} \\ \gamma_{yz} \\ \gamma_{yz} \end{cases} - \begin{cases} \sigma_{cz_{x}}^{o} \\ \sigma_{cz_{x}}^{o} \\ \tau_{cz_{xy}}^{o} \\ \tau_{cz_{xy}}^{o} \\ \tau_{cz_{yz}}^{o} \\ \tau_{cz_{yz}}^{c$$

Similarly, steel layer stresses are calculated as:

$$\{\sigma_s\} = [D_s]\{\varepsilon\} - \{\sigma_s^o\}$$
(5-48)

where the steel layer pseudo-stress vector,  $\{\sigma_s^o\}$ , is calculated from:

$$\left\{\sigma_{s}^{o}\right\} = \left[D_{s}\right]\left(\left\{\varepsilon_{s}^{o}\right\} + \left\{\varepsilon_{s}^{p}\right\}\right)$$
(5-49)

### 5.3.2 Enforcing Zero Normal Stress

To enforce the zero normal stress condition of the shell element, two modifications are applied in the solution of the concrete layers: 1) modification of the combined concrete material matrix,  $[D_{cz}]$ , and 2) modification of the developed concrete pseudo-stress vector,  $\{\sigma_{cz}^o\}$ .

The expanded equilibrium equation for a concrete layer can be expressed as:

$$\begin{cases} \sigma_{cz_{x}} \\ \sigma_{cz_{y}} \\ \sigma_{cz_{x}} \\ \tau_{cz_{xy}} \\ \tau_{cz_{xz}} \\ \tau_{cz_{yz}} \end{cases} = \begin{bmatrix} D_{cz_{11}} & D_{cz_{12}} & D_{cz_{13}} & D_{cz_{14}} & D_{cz_{15}} & D_{cz_{16}} \\ D_{cz_{11}} & D_{cz_{22}} & D_{cz_{23}} & D_{cz_{24}} & D_{cz_{25}} & D_{cz_{26}} \\ D_{cz_{31}} & D_{cz_{32}} & D_{cz_{33}} & D_{cz_{34}} & D_{cz_{35}} & D_{cz_{36}} \\ D_{cz_{41}} & D_{cz_{42}} & D_{cz_{43}} & D_{cz_{44}} & D_{cz_{45}} & D_{cz_{46}} \\ D_{cz_{51}} & D_{cz_{52}} & D_{cz_{53}} & D_{cz_{54}} & D_{cz_{55}} & D_{cz_{56}} \\ D_{cz_{61}} & D_{cz_{62}} & D_{cz_{63}} & D_{cz_{64}} & D_{cz_{65}} & D_{cz_{66}} \end{bmatrix} \begin{pmatrix} \varepsilon_{x} \\ \varepsilon_{y} \\ \varepsilon_{z} \\ \varepsilon_{z} \\ \varepsilon_{z} \end{pmatrix} = \begin{cases} \sigma_{cz_{x}}^{o} \\ \sigma_{cz_{x}}^{o} \\ \varepsilon_{z} \\ \varepsilon_{$$

As the total out-of-plane strain,  $\varepsilon_z$ , is not directly calculated from the strain-displacement relationship of the element, the assumption that  $\sigma_{cz}$  is negligible (i.e.,  $\sigma_{cz} = 0$ ) results in:

$$\varepsilon_{z} = \frac{\sigma_{cz_{z}}^{o} - D_{cz_{31}} \cdot \varepsilon_{x} - D_{cz_{32}} \cdot \varepsilon_{y} - D_{cz_{34}} \cdot \gamma_{xy} - D_{cz_{35}} \cdot \gamma_{xz} - D_{cz_{36}} \cdot \gamma_{yz}}{D_{cz_{33}}}$$
(5-51)

To enforce the zero out-of-plane stress condition in the concrete material matrix,  $[D_{cz}]$ , the columns and rows pertaining to the z' direction (i.e., row 3 and column 3) are removed by applying Equation 5-52.

$$D_{cz_{ij}}^* = D_{cz_{ij}} - \frac{D_{cz_{i3}} D_{cz_{3j}}}{D_{cz_{33}}}$$
(5-52)

The resulting material matrix,  $[D_{cz}^*]$ , is of 5 x 5, and is no longer dependant on  $\varepsilon_z$  or  $\sigma_{cz}$ . To accommodate the strain offsets in the concrete and the out-of-plane steel, the pseudo-stress vector must also be modified in a similar fashion:

$$\sigma_{cz_{i}}^{*o} = \sigma_{cz_{i}}^{o} - \frac{D_{cz_{i3}}\sigma_{cz_{i}}^{o}}{D_{cz_{33}}}$$
(5-53)

The modified equilibrium equation then takes a form that matches with the formulation of the shell finite element (see Section 5.1.6):

$$\begin{cases} \sigma_{cz_{x}} \\ \sigma_{cz_{y}} \\ \tau_{cz_{xy}} \\ \tau_{cz_{xz}} \\ \tau_{cz_{yz}} \end{cases} = \begin{bmatrix} D_{cz}^{*} \end{bmatrix}_{5x5} \begin{cases} \varepsilon_{x} \\ \varepsilon_{y} \\ \gamma_{xy} \\ \gamma_{xy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{cases} - \begin{cases} \sigma_{cz_{x}}^{o^{*}} \\ \sigma_{cz_{yy}}^{o^{*}} \\ \tau_{cz_{xy}}^{o^{*}} \\ \tau_{cz_{xy}}^{o^{*}} \\ \tau_{cz_{yz}}^{o^{*}} \\ \tau_{cz_{yz}}^{o^{*}} \end{bmatrix}$$
(5-54)

#### 5.3.3 Solution Algorithm

The solution method adopted in VecTor4 uses a total load, secant stiffness approach. The structural stiffness matrix, [K], is evaluated using secant stiffness properties computed from the secant material matrices developed in Section 5.3.1. Consideration of full loads acting on the structure are assembled in the total nodal load vector,  $\{F\}$ , and the resulting nodal displacement vector,  $\{\delta\}$ , represents the total structure displacements under  $\{F\}$ . Expressed generally:

$$\{\delta\} = [K]^{-1}\{F\}$$
(5-55)

The global stiffness matrix of the structure is developed through assembly of the elemental stiffness matrices,  $[k_e]$ . Similarly, elemental load vectors,  $\{f_e\}$ , are used to assemble the global nodal load vector:

$$\begin{bmatrix} K \end{bmatrix} \leftarrow \sum_{e} [k_e] \tag{5-56a}$$

$$\{F\} \leftarrow \sum_{e} \{f_e\} \tag{5-56b}$$

where the elemental stiffness matrices and load vectors are defined as:

$$[k_e] = \int_{vol} [B]^T [D] [B] dv$$
(5-56a)

$$\{f_e\} = \int_{vol} [B]\{\sigma\} dv \tag{5-56a}$$

The consideration of pseudo-stresses resulting from the various offsets discussed in Section 5.3.1, are accommodated by way of prestress load vectors denoted by  $\{f_e^o\}$ , where the prestress vectors are calculated in the same manner as the elemental load vectors:

$$\left\{f_e^o\right\} = \int_{vol} \left[B\right] \left\{\sigma^o\right\} dv \tag{5-57}$$

The inclusion of the structure prestress load vector,  $\{F^o\}$ , results in the revised equilibrium expression presented in Equation 5-58.

$$\{\delta\} = [K]^{-1} (\{F\} + \{F^o\})$$
(5-58)

Because of the nonlinear nature of the problem, the unknown nodal displacement values,  $\{\delta\}$ , are required to develop the global stiffness matrix, [K]. As such, an iterative solution procedure is adopted. For a given load step or increment, the iterative procedure is repeated until the solution has converged within a predefined error limit, or a predefined maximum number of iterations has been performed. By default, VecTor4 evaluates convergence based on the change in the nodal displacement values computed from one iteration to the next.

The main steps of the global solution algorithm are presented in Figure 5.17. Note that to enhance solution stability and to minimize the number of global iterations required to achieve convergence, a double-iterative solution procedure is employed.



Figure 5.17 - VecTor4 Solution Algorithm

# CHAPTER 6: MONOTONIC LOADING

This chapter examines the application and effectiveness of VecTor4 in analyzing different types of structures subjected to monotonic loading conditions. Key additions to the program which have been developed through this work are summarized, and their implementation is verified through the performance of an extensive verification study.

Structures considered in the monotonic verification include: i) shear-critical beams, with and without out-of-plane shear reinforcement (Vecchio and Shim, 2004); ii) shear-critical thick slabs, with and without out-of-plane shear reinforcement (Jaeger and Marti, 2009a); iii) thin-plate elements subjected to combined in-plane and out-of-plane loads (Aghayere and MacGregor, 1990); iv) slab and beam elements subjected to combined tension and out-of-plane shear (Mattock, 1969; Leonhardt et al., 1977; Sørenson et al., 1981); v) reinforced concrete shell elements subject to combined in-plane shear and out-of plane shear and combined in-plane shear and bending (Adebar and Collins, 1991; Polak and Vecchio, 1994); and lastly, vi) steel fibre reinforced concrete (SFRC) panel elements containing conventional reinforcement (R/FRC) subjected to pure in-plane shear (Susetyo et al., 2011).

The chapter concludes with a general overview of the results obtained from the verification study. Successes and limitation are noted, and recommendations for further study are presented.

### 6.1 Monotonic Loading: Development and Implementation

This section of the thesis presents some of the main developments and features that have been added to VecTor4. The presented program additions and modifications represent general features which, in many cases, contribute significantly to the analytical results generated from the program. The implementation of these features was motivated by specific analysis needs or deficiencies identified throughout the process of verifying the program's performance.

### 6.1.1 Local Conditions at the Crack

In previous versions of VecTor4 (programs APECS and RASP), a simplified methodology was adopted for the evaluation of the local conditions at the crack. Average concrete tensile stresses were appropriately limited based on the reinforcement's ability to carry the stresses across the crack; however, no consideration was given to the local shear stresses acting on the crack surface.

The local crack-slip check is a key feature of the MCFT and its inclusion in VecTor4 is essential. Additionally, slip deformation behaviour considered in the formulations of the DSFM stem directly from the development of the shear stresses on the crack surface. This section presents a general overview of the analysis methodology considered in VecTor4 to evaluate the local behaviour at the crack location.

As noted in the previous chapter, reinforced concrete may carry appreciable tensile stresses between cracks through tension stiffening mechanisms. Currently, several tension stiffening models are available in VecTor4; they are used to calculate average concrete tensile stresses from a given state of average concrete strains. The original formulation of the MCFT considered the following relationship:

$$f_{c1} = \frac{f_t'}{1 + \sqrt{200\varepsilon_{c1}}}$$
(6-1)

where,

 $f_{cl}$  = the average concrete tensile stress in the principal 1-direction, (MPa);

 $f_t$  '= the tensile strength of the concrete, (MPa);

 $\varepsilon_{cl}$  = the average concrete tensile strain in the principal 1-direction, (mm/mm);

To ensure that the reinforcement is capable of carrying the concrete tensile stresses across the cracks, the following limitation is considered:

$$f_{c1} \le f_{c1}^* = \sum_{i=1}^n \rho_i (f_{yi} - f_{si}) \cos^2 \theta_{1i} \quad ; \quad \cos \theta_{1i} = l_1 l_i + m_1 m_i + n_1 n_i$$
(6-2)

where,

 $f_{c1}^*$  = the maximum tensile stress which can be carried across the crack, (MPa);  $\rho_i$  = the reinforcement ratio corresponding to reinforcement component *i*;  $f_{yi}$  = the yield stress of reinforcement component *i*, (MPa);
$f_{si}$  = the average stress in reinforcement component *i*, (MPa);

 $\theta_{li}$  = the angle between the reinforcement direction and the principal 1-direction, (rads, degrees);

- $l_1, m_1, n_1$  = the direction cosines of a unit vector in the principal 1-direction;
- $l_i, m_i, n_i$  = the direction cosines of a unit vector in the direction of reinforcement component *i*.

An arbitrarily cracked reinforced concrete element which contains reinforcement in only one direction (denoted as reinforcement component *i*) is presented in Figure 6.1. To illustrate the generalized approach used in VecTor4 to analyze crack surfaces of three-dimensional elements, it is assumed that reinforcement component *i* is not oriented in any of the principal planes (i.e., plane 1-2, plane 1-3, or plane 2-3). Examination of the principal planes shows that for cracking in the 1-direction, average concrete stresses  $f_{c2}$  and  $f_{c3}$  (both of which were arbitrarily assumed to be compressive in this sample case) have no influence on the local stresses at the crack.

To maintain equilibrium in the principal 1-direction, the development of increased stress in the reinforcement across the crack surface, denoted as  $f_{sicr}$ , are required to balance the average concrete tensile stress,  $f_{c1}$ . Equilibrium in the principal 2- and 3-directions requires that shear stresses on the crack surface balance the stresses developed in the 2- and 3-directions as a result of the localized increase in reinforcement stress.



Figure 6.1 – 3D Local Crack Surface

For an element cracked in the 1-direction, the shear on the crack in the principal planes ( $v_{ci,12}$  and  $v_{ci,13}$ ) can be calculated from:

$$v_{ci,12} = \sum_{i=1}^{n} \rho_i (f_{sicr} - f_{si}) (l_i l_1 l_2 + m_i m_1 m_2 + n_i n_1 n_2)$$
(6-3a)

$$v_{ci,13} = \sum_{i=1}^{n} \rho_i (f_{sicr} - f_{si}) \left( l_i l_1 l_3 + m_i m_1 m_3 + n_i n_1 n_3 \right)$$
(6-3b)

where,

 $f_{sicr}$  = the local stress across the crack in reinforcement component *i*, (MPa);  $l_2, m_2, n_2$  = the direction cosines of a unit vector in the principal 2-direction;  $l_3, m_3, n_3$  = the direction cosines of a unit vector in the principal 3-direction; all other variables are as defined above in Equation 6-2.

It is important to note that because some reinforcement contributions will result in additive shear stresses on the crack surface, and others will reduce the crack shear stress, care must be taken in evaluating Equation 6-3, particularly with respect to the direction cosines used to define the reinforcement orientation  $(l_i, m_i, n_i)$ .

The magnitude of the resultant shear stress on the crack surface in the 1-direction,  $v_{ci,1}$  can then be calculated as the vector sum of the shears in the principal planes:

$$v_{ci,1} = \sqrt{v_{ci,12}^{2} + v_{ci,13}^{2}}$$
(6-4)

In the application of the MCFT, the resultant shear stress on the crack is compared with allowable stress limits ( $v_{ci} \le v_{ci,max}$ ), and if exceeded, requires reduction of the average tensile stress. The DSFM uses the shear stresses on the crack surface to calculate slip deformations and average element slip strains which are considered in element compatibility.

An iterative procedure is adopted in VecTor4 to evaluate the behaviour of the local crack surface. The following algorithm is used to evaluate the local behaviour for an element cracked in the principal 1-direction:

- i. estimate the strain across the crack in the principal direction:  $\varepsilon_{1,cr} = \varepsilon_1 + \Delta \varepsilon_{1,cr}$ ;
- ii. calculate the stresses in the reinforcement across the cracks:  $f_{scri}$ ;
- iii. calculate the tensile stress transferred across the crack:  $f_{c1}' = \sum_{i=1}^{n} \rho_i (f_{scri} f_{si}) \cos^2 \theta_{1i}$ ;
- iv. check equilibrium in 1-direction:  $f_{cl} = f_{cl}$ ? If not, return to step i, revise estimate of  $\varepsilon_{1,cr}$ ;
- v. calculate  $v_{ci,12}$ ,  $v_{ci,13}$ , and  $v_{ci,1}$ ;
- vi. (MCFT) if  $v_{ci,1} > v_{ci,max}$ , apply linear reduction to  $f_{c1}$ :  $f_{c1} = f_{c1} \left( \frac{v_{ci,max}}{v_{ci,1}} \right)$ ;
- vii. (DSFM) use  $v_{ci,1}$  to calculate the crack-slip deformation,  $\delta_s$ , and the average slip strain,  $\gamma_s$ ;
- viii. (DSFM) calculate local slip strain vector  $\{\varepsilon^s\}$  from  $\gamma_s$ , with reference to the orientation of  $v_{ci,12}$  and  $v_{ci,13}$ .

The above procedure is considered for all cracked principal directions. It is important to note that concrete cracked in two, or even three, orthogonal directions is not uncommon, particularly for structures subjected to cyclic or dynamic loading conditions.

### 6.1.2 Out-of-Plane Shear

Perhaps the most significant contribution made to the analysis procedure of VecTor4 pertains to how out-of-plane shear is considered. As noted in the previous chapter, out-of-plane shear deformations are calculated directly from the element displacement field by way of the inherent 'thick-shell' formulation. Out-of-plane shear strains are computed from nodal displacements and represent the sectional shear deformation. As a result, the out-of-plane shear strains are naturally treated as being constant and uniform through the depth of the section (see Figure 6.2).



Figure 6.2 – Assumed Strain Distribution; (a) Section A-A; (b) longitudinal strains; (c) uniform shear strains

In a preliminary assessment of VecTor4's ability to model simple beam elements subjected to out-of-plane shear and flexure, it was found that the use of the uniform out-of-plane shear strain distribution typically resulted in a significant overestimation of the out-of-plane shear strength, particularly in the case of shear-critical elements. The uncracked compressive regions of the beams were found to carry artificially high shear stresses and, as a result, allowed the beams to resist the full shear demands over depths as small as 10 % of the section heights. Additionally, the largest overestimations in shear strengths were found to occur in members containing no out-of-plane shear reinforcement which was most concerning given that many practical RC shell structures are comprised of elements with little or no out-of-plane shear reinforcement.

Other researchers employing MCFT/DSFM based sectional analysis methods have demonstrated success in modelling out-of-plane shear using a broad range of methods with the simplest approaches involving assumed out-of-plane shear strain or shear flow distributions (Vecchio and Collins, 1988 ; Guner and Vecchio, 2010a,b) and the more complex methods considering rigorous dual-section analyses or the application of explicit numerical techniques to calculate the out-of-plane shear stress and strain profiles (Vecchio and Collins, 1988 ; Bentz, 2000). While, of those noted above, the more complex methods have demonstrated superior accuracy over approximate methods, they are significantly more computation intensive and, more importantly, are not well-suited for finite elements which are typically formulated on the basis of some shear deformation assumption (e.g., Mindlin theory in the case of VecTor4). The more complex techniques also present additional challenges in the performance of displacement-controlled, cyclic, and dynamic analyses.

The application of an assumed parabolic shear strain distribution through the depth was investigated by Vecchio and Collins (1988) and was shown to reasonably capture the behaviour of RC beams subjected to combined axial loading, shear, and moments. It was reported that although the approximate method was much more stable and substantially less computationally demanding than other methods, the parabolic strain distribution had the tendency to overestimate the shear stresses in the compressive regions of the beams, and in some cases resulted in unconservative strength estimates. However, because one of the primary goals of this study was to develop cyclic and dynamic analysis capabilities within VecTor4, the stability of the assumed shear strain distribution warranted the simplified approach.

A vast amount of research has been performed in an effort to improve upon the first-order shear deformation (FSDT) theory (i.e., uniform out-of-plane shear strain) typically employed for layered finite elements. The research can generally be divided into two categories: development of higher-order shear deformation theories (HSDT) which serve as a replacement of first-order theory, and the development of modification methods which are used in conjunction with first-order theory, also referred to as modified first-order shear deformation theories (MFSDT). An extensive review of several HSDT theories is provided by Ghugal and Shimpi (2001).

In the case of higher-order theories, non-uniform out-of-plane shear strains arise from the use of higher-order equations describing the displacement field of the finite element. The benefits of such an approach is that shear strains are calculated directly from nodal displacements without further calculation, and troublesome shear-locking phenomena discussed in the previous chapter are removed from the problem entirely. The main disadvantage of using higher-order theories is that many of them require the solution of additional degrees of freedom to represent sectional warping, and these theories also deviate from the well-accepted plane sections assumption.

Simple modifications to the first-order shear deformation theory which incorporate non-uniform out-of-plane shear strain distributions and still consider plane sections behaviour have been proposed by some (Tanov and Tabiei, 2000; Han et al., 2008). However, the use of such methods has been primarily limited to the analysis of thin laminar-composite elements, and it has been suggested that the methods involving partial modification of the element strain-displacement relations, as is done in Tanov and Tabiei (2000), may result in appreciable error for other types of structural elements (Han et al., 2008).

To consider non-uniform out-of-plane shear strain distributions in VecTor4, a method similar to that of the modified first-order type presented by Tanov and Tabiei (2000) was considered; however, some notable differences were introduced. In most modified first-order formulations, alteration of the strain-displacement relations form the basis of the imposed modifications; however, in VecTor4, out-of-plane shear strain modifications are incorporated entirely within the material stiffness matrix,  $[D]^*$ . Note that the following equations are based on the assumption of a parabolic shear strain distribution, but can be recast to accommodate any desired/assumed shear strain distribution.

As discussed in Chapter 5, the local strain vector pertaining to a single integration point within layer *i* (see Figure 6.2) is evaluated on the basis of the element nodal displacements,  $\{\delta\}$ , and the strain-displacement matrix,  $[B_i]$ :

$$\{\varepsilon_i\} = [B_i]\{\delta\} = \left\langle \varepsilon_{x,i} \quad \varepsilon_{y,i} \quad \gamma_{xy,i} \quad \gamma_{xz,i} \quad \gamma_{yz,i} \right\rangle$$
(6-5)

The out-of-plane shear strains are modified to account for the parabolic distribution using Equation 6-6:

$$\gamma_{xz,i}^* = \gamma_{xz,i} \left( 1 - \zeta_i^2 \right) \tag{6-6a}$$

$$\gamma_{yz,i}^* = \gamma_{yz,i} \left( 1 - \zeta_i^2 \right) \tag{6-6b}$$

where,

{

 $\gamma_{xz,i}$ ,  $\gamma_{yz,i}$  = out-of-plane shear strains calculated from the element displacement field, (mm/mm);  $\gamma^*_{xz,i}$ ,  $\gamma^*_{yz,i}$  = modified out-of-plane shear strains, (mm/mm);  $\zeta_i$  = location of layer *i* within the depth of the shell element (see Figure 6.2).

The material matrix,  $[D_i]$  is then developed on the basis of the MCFT/DSFM behavioural models using the modified strain vector,  $\{\varepsilon_i\}^*$ :

$$\left\{\varepsilon_{i}\right\}^{*} = \left\langle\varepsilon_{x,i} \quad \varepsilon_{y,i} \quad \gamma_{xy,i} \quad \gamma_{xz,i}^{*} \quad \gamma_{yz,i}^{*}\right\rangle$$
(6-7)

The resulting equilibrium relation is presented in Equation 6-8a. Further modification of the material matrix is performed such that the local stresses are calculated on the basis of the original local strain vector (see Equations 6-8b and 6-9).

$$\{\sigma_i\} = [D_i]\{\varepsilon_i\}^* - \{\sigma_i^o\}$$
(6-8a)

$$\sigma_i\} + \left\{\sigma_i^o\right\} = \left[D_i\right] \left\{\varepsilon_i\right\}^* = \left[D_i\right]^* \left\{\varepsilon_i\right\}$$
(6-8b)

where the modified material matrix is calculated from:

$$\begin{bmatrix} D_i \end{bmatrix}^* = \begin{bmatrix} D_i \end{bmatrix} \begin{cases} 1 \\ 1 \\ 1 \\ (1 - \zeta^2) \\ (1 - \zeta^2) \end{cases}$$
(6-9)

As a result, the global stiffness matrix contributions become a function of the modified material matrix,  $[D]^*$ , and the original strain-displacement relations of the shell element, [B]:

$$\begin{bmatrix} k_e \end{bmatrix} = \int_{vol} \begin{bmatrix} B \end{bmatrix}^T \begin{bmatrix} D \end{bmatrix}^* \begin{bmatrix} B \end{bmatrix} dv$$
(6-10)

This out-of-plane strain modification method ensures that equilibrium is always preserved, with sectional forces equaling those calculated from global equilibrium relations. Additionally, the method is simple to implement, and it results in no significant additional computational cost.

To illustrate the significance of the modified shear strain distribution, consider the VecTor4 computed sectional responses for Beam VS-OA1 (see Figure 6.3). Note that the selected beam contains no out-of-plane shear reinforcement and forms part of an experimental series of beams tested by Vecchio and Shim (2004). Details of the testing program can be found in Section 6.2.1.

	longitudinal strain $(\mathcal{E}_{\mathbf{x}'} \times 10^{-3} \text{ mm/mm})$	out-of-plane normal strain (E-' x 10 ⁻³ mm/mm)	out-of-plane shear strain ( $\gamma_{r'z'}$ x 10 ⁻³ mm/mm)	out-of-plane shear stress $(\tau_{x'x'}$ MPa)
uniform out-of-plane shear strain	-0.63	0.098	0.19	1.54
parabolic out-of-plane shear strain	-0.63	0.098	0.37	2.69

Figure 6.3 – Computed Sectional Behaviour for Beam VS-OA1;  $M_x = 155$  kN-m,  $V_{xz} = 150$  kN

From Figure 6.3 it can be seen that the out-of-plane sectional behaviour of the beam, both normal and shear, is significantly influenced by the assumed out-of-plane shear strain distribution. Under the same loading conditions, the parabolic shear strain profile yields reduced shear stresses in the compressive layers of the section, increased shear strains at the mid-depth of the section, reduced shear strains at the extreme fibres of the section, and increased out-of-plane normal strains throughout the cracked layers of the section.

The computed load deflection behaviours for Beam VS-OA1 resulting from the two shear strain profiles considered are presented in Figure 6.4a. It can be seen that the modified out-of-plane shear strain distribution estimates a significantly lower shear capacity than the uniform distribution, and agrees well with the behaviour observed experimentally.

In Figure 6.4b, the computed responses for Beam VS-A1 have been provided. This beam was nominally identical to OA1; however, it contained 0.10 % out-of-plane shear reinforcement. It can be seen that for the beam containing shear reinforcement, the estimated beam capacity only marginally differs as a result of the assumed shear strain profile. However, the computed stiffness and ductility are significantly impacted, with much better agreement being achieved with the use of the parabolic shear strain profile.



Figure 6.4 – Influence of Shear Strain Profile on Computed Response

The use of an approximate parabolic out-of-plane shear strain profile is found to significantly improve the computed responses of shear-critical elements subjected to out-of-plane shear. The method used to modify the naturally uniform strain condition in the VecTor4 thick-shell element requires no significant additional computation, and has no impact on program stability. Generally, the use of the parabolic profile over a uniform strain profile in members subjected to combined flexure and shear results in a decrease of the sectional shear stiffness.

## 6.1.3 Disturbed Regions

Typical structures which are subjected to flexural loading can be viewed as being comprised of a combination of disturbed and undisturbed regions. Undisturbed regions, which are commonly referred to as B-regions (where B is in reference to Bernoulli or beam), represent the locations of a structure where Bernoulli's plane sections hypothesis is believed to be a sufficiently accurate approach for describing the in-plane sectional strain distribution. Disturbed regions, which are commonly referred to as D-regions (where D refers to discontinuity, disturbance, or detail), form the regions of the structure where the strain distribution may be highly nonlinear and Bernoulli's hypothesis is not valid. Examples of D-regions include locations of concentrated loading, support restraint locations, and abrupt changes in element geometry (Schlaich et al., 1987).

If present, the types of disturbances noted above can significantly influence the sectional behaviours within these regions. The occurrence of direct strut action from load to support, or the addition of out-of-plane confining stresses introduced at the location of a concentrated load or support, will not only effect the in-plane strain distribution but can also lead to significant shear strength enhancement within these regions. As such, modelling D-regions using sectional analyses employing plane sections behaviour may result in overly conservative results.

Several methods have been used to incorporate strength enhancements resulting from disturbed regions into sectional analysis procedures (see Figure 6.5). For example, the Canadian design provisions (CSA A23.3-04) permit that one-way sectional analyses be performed at a distance  $d_v$  ( $d_v$  taken as the minimum of 0.72*h* or 0.9*d*) away from the edge of a concentrated load or joint face for the purposes of assessing shear strength or designing shear reinforcement. In the MCFT based sectional analysis program Response2000 (Bentz, 2000), the applied shear forces acting on the member are linearly reduced over a distance of *d* from the support and loading locations. The

software program VecTor5, which is based on the formulations of the DSFM, reduces the effective shear force by 50 % over a distance of 0.70*h* measured from the face of the adjoining member, support, or load location.



Figure 6.5 - 'Active' Shear Force Distributions

Based upon the results from a series of verification studies presented in the following sections of this chapter, the analytical responses for structures subjected to monotonic loading conditions were determined to be sufficiently accurate without considering disturbed region strength enhancements. However, it has been found that for elements which develop high levels of plastic strain and damage resulting from repeated and reversed loading scenarios (i.e., cyclic and dynamic loading conditions), VecTor4 has shown the tendency to underestimate the strength and ductility of elements forming the disturbed regions of a structure. As such, in VecTor4 disturbed regions are not subject to any form of strength enhancement by default; however, the option to consider out-of-plane shear strength enhancements has been included in the program.

Shear strength enhancement in VecTor4 is achieved through the reduction of the apparent, or active, calculated out-of-plane shear strains within a specified element. The level of reduction and the affected elements are both user-specified, and the reductions are incorporated using a simple factor:

$$\gamma'_{xz(yz)} = \gamma_{xz(yz)} \cdot SSM \tag{6-11}$$

where,

 $\gamma'_{xz(yz)}$  = reduced out-of-plane shear strain, (mm/mm); SSM = shear strain multiplier, (user specified factor ranging from zero to 1.0). The concrete material matrix is then computed on the basis of the user-defined reduced out-ofplane shear strains,  $\gamma'_{xz}$  and  $\gamma'_{yz}$ .

As discussed above, several approaches have been used to address shear strength enhancement for disturbed regions in sectional analysis programs and provisions. Because shear strength enhancement in VecTor4 is user-defined, any 'active' shear force distribution can be achieved (see Figure 6.5). However, due to the simplicity of the method and the relatively large element sizes which are typically employed in the meshes of VecTor4, it is convenient to use a method similar to that considered in the sectional analysis program VecTor5; that is, a constant reduction over some user defined distance or area.

## 6.1.4 Steel Fibre Reinforced Concrete (SFRC)

One of the initial goals identified in this research program was to incorporate a behavioural model within the VecTor4 solution algorithm which could be used to model SFRC structures. There are many models available in the literature which could potentially serve as viable approaches for analyzing SFRC using VecTor4; however, currently only one model has been implemented.

The Simplified Diverse Embedment Model (SDEM), developed by Lee et al. (2013a), is a rational model which can be used to describe the behaviour of concrete containing a random distribution of discontinuous steel fibres subjected to uniaxial tensile stress. The model was developed as a simplification of the original formulation, the Diverse Embedment Model (DEM), presented by the same authors (Lee et al., 2011a). The primary advantage of the SDEM over the originally developed DEM is the elimination of the double numerical integration required in the solution algorithm.

In brief, the SDEM computes average fibre tensile stresses across the cracks as the summation of stresses developed from frictional bond and stresses developed from mechanical anchorage effects. The inclusion of the anchorage contributions, such as those provided by end-hooked steel fibres, distinguishes the model from many others available in the literature. Consideration of random fibre inclination and random fibre embedment length form the basis of the relationships

developed. The complete development and formulation of the SDEM fibre tension model is presented in (Lee et al., 2013a); however, a short summary of the relationships forming the fibre tension model are presented.

## SDEM Fibre Tension Model

For both end-hooked and straight steel fibres, the average fibre tensile stress developed by way of frictional bond,  $f_{st}$ , is calculated from:

$$f_{st} = \alpha_f V_f K_{st} \tau_{f,\max} \frac{l_f}{d_f} \left( 1 - \frac{2w_{cr}}{l_f} \right)^2$$
(6-12a)

and

$$K_{st} = \begin{cases} \frac{\beta_f}{3} \frac{w_{cr}}{s_f} & \text{for } w_{cr} < s_f \\ 1 - \sqrt{\frac{s_f}{w_{cr}}} + \frac{\beta_f}{3} \sqrt{\frac{s_f}{w_{cr}}} & \text{for } w_{cr} \ge s_f \end{cases}$$
(6-12b)

$$\tau_{f,\max} = 0.396\sqrt{f_c}$$
(6-12c)

where,

 $\alpha_f$  = fibre orientation factor (taken as 0.50 for a 3D infinite element) (Aveston and Kelly, 1973);

 $V_f$  = fibre volume fraction;

$$K_{st}$$
 = bond modulus, (MPa);

 $\tau_{f,\text{max}}$  = fibre pullout strength (from concrete), (MPa);

 $l_f$  = fibre length, (mm);

 $d_f$  = fibre diameter, (mm);

 $w_{cr} = \text{crack width}, (\text{mm});$ 

 $s_f$  = slip at the frictional bond strength for a fiber oriented perpendicular to the crack, (mm); (assumed to be 0.01 mm)

 $\beta_f$  = calibration factor to fit results to full DEM ( $\beta_f$  = 0.67).

The tensile stress contribution from the mechanical anchorage of end-hooked steel fibres,  $f_{eh}$ , is calculated from:

$$f_{eh} = \alpha_f V_f K_{eh} \tau_{eh,\max} \frac{2(l_i - 2w_{cr})}{d_f}$$
(6-13a)

and

$$K_{eh} = \begin{cases} \beta_{eh} \left[ \frac{2}{3} \frac{w_{cr}}{s_{eh}} - \frac{1}{5} \left( \frac{w_{cr}}{s_{eh}} \right)^2 \right] & \text{for } w_{cr} < s_{eh} \\ 1 + \left( \frac{7\beta_{eh}}{15} - 1 \right) \sqrt{\frac{s_{eh}}{w_{cr}}} - \frac{2\left( \sqrt{w_{cr}} - \sqrt{s_{eh}} \right)^2}{l_f - l_i} & \text{for } s_{eh} \le w_{cr} < \frac{l_f - l_i}{2} \\ \left( \frac{l_i - 2w_{cr}}{2l_i - l_f} \right)^2 K_{eh,i} & \text{for } \frac{l_f - l_i}{2} \le w_{cr} < \frac{l_i}{2} \end{cases}$$
(6-13b)

$$\tau_{eh,\max} = 0.429 \sqrt{f_c}$$
 (6-13c)

where,

 $K_{eh}$  = anchorage modulus, (MPa);

$$K_{eh,i} = K_{eh}$$
 at  $w_{cr} = (l_f - l_i)/2$ , (MPa);

 $l_i$  = distance between end-hooks of fibre, (mm);

 $\tau_{eh,max}$  = pull-out strength resulting from mechanical anchorage of an end-hooked fiber, (MPa);

- $s_{eh}$  = slip at the maximum anchor strength for a fiber oriented perpendicular to the crack, (mm); (assumed to be 0.1 mm)
- $\beta_f$  = calibration factor to fit results to full DEM ( $\beta_f$  = 0.76).

The full tensile stress attained by the steel fibres,  $f_f$ , is then calculated from the summation of bond and anchorage contributions:

$$f_f = f_{st}$$
 (for straight fibres) (6-14a)  
 $f_f = f_{st} + f_{eh}$  (for end-hooked fibres) (6-14b)

### Local Crack Conditions

As summarized in the above presentation of the SDEM fibre-tension model, the addition of steel fibres will result in the development of increased tensile stresses across the crack. Additionally, however, steel fibre contributions to the concrete crack-slip resistance should also be considered in the implementation of the model.

In the event that local slip deformations occur on the crack surface, the steel fibre stress across the crack will deviate from the normal of the crack surface (see Figure 6.6). As a result of this deviation, the addition of steel fibres can lead to significant reductions in the crack-slip deformations and, in turn, increased local crack-slip resistance. The following procedure has been adopted for evaluating the FRC contribution to the shear stress on the surface of a crack oriented perpendicular to the principal 1-direction. Note that the fibre tension stress transferred across the crack is denoted as  $f_{f_5}$  and the average fibre stress contribution to the concrete between the cracks is denoted as  $\varphi f_{f_5}$  where the factor  $\varphi$  is developed from the steel fibre stress attenuation (Lee et al., 2011b) and is a function of the fibre length,  $l_{f_5}$  and the average crack spacing,  $s_{cr}$ .



Figure 6.6 – 3D Local Equilibrium, SFRC

Equation 6-15 is used to calculate the total steel fibre displacement across the crack,  $w_{cr}$ . The fibre displacement is considered to be the resultant of the fibre deformations occurring from

crack opening, and displacements which occur from crack-slip,  $\delta^s$ . Note that the methodology used to calculate slip deformations has been outlined in Section 6.1.

$$w_{cr}' = \sqrt{w_{cr}^2 + (\delta^s)^2}$$
(6-15)

The total crack-slip deformation on the local crack surface is decomposed into slip deformations in the principal 2- and principal 3-directions from the following:

$$\delta_{12}^{s} = \delta^{s} \left( \frac{v_{ci,12}}{v_{ci,1}} \right) \qquad ; \qquad \delta_{13}^{s} = \delta^{s} \left( \frac{v_{ci,13}}{v_{ci,1}} \right) \tag{6-16}$$

The direction cosine vector  $\psi_f$  describes the orientation of the steel fiber stress across the crack surface, relative to the directions of principal concrete stress:

$$\Psi_f = \left\langle \frac{W_{cr}}{W_{cr}}, \frac{\delta_{12}^s}{W_{cr}}, \frac{\delta_{13}^s}{W_{cr}} \right\rangle$$
(6-17)

Lastly, the angle formed between the slip direction on the crack surface and the orientation of the steel fibre stress,  $\theta_f$ , is computed as:

$$\theta_f = \tan^{-1} \left( \frac{\delta^s}{w_{cr}} \right) \tag{6-18}$$

Referring to Equations 6-3 and 6-4, the reduction of shear stress on the crack is then calculated using the following:

$$v_{ci,1} = \sqrt{v_{ci,12}^{2} + v_{ci,13}^{2}} - (1 - \varphi)f_f \sin\theta_f$$
(6-19)

where  $(1-\varphi)f_f$  represents the increase in fibre stress across the crack.

### Contribution to the Composite Material Matrix

In a similar fashion to that of the steel and concrete contributions to the composite material matrix, the secant modulus of the FRC across the crack in the principal *i*-direction is calculated from the average fibre stress in the concrete between the cracks and the net concrete strains:

$$\overline{E}_{f,i} = \frac{\varphi f_f}{\varepsilon_{cf,i}}$$
(6-20)

and

$$\varepsilon_{cf,i} = \varepsilon_{c1} \cdot \psi_{f,i1} + \varepsilon_{c2} \cdot \psi_{f,i2} + \varepsilon_{c3} \cdot \psi_{f,i3}$$
(6-21)

where,  $\psi_{f,i1}$ ,  $\psi_{f,i2}$ , and  $\psi_{f,i3}$  are the direction cosines describing the fibre orientation across crack perpendicular to the principal *i*-direction (calculated from Equation 6-17). The local material matrix for the FRC is expressed as:

The local FRC material matrix,  $[D_{f,i}]$ ', is then transformed to the principal stress directions of the concrete using the direction cosines defined in Equation 6-17. The fibre contributions are then combined with the local concrete material matrix, and are transformed to global system using the methodology presented in Chapter 5.

#### Tension Stiffening

By default, VecTor4 considers a modified version of the tension stiffening model proposed by Bentz (2005). This model is similar in form to that originally proposed by Vecchio and Collins (Vecchio and Collins, 1986); however, it considers additional factors pertaining to the amount, and characteristics, of the reinforcement in the concrete.

The model takes the following general form (Wong et al., 2012):

$$f_{c1} = \frac{f_t'}{1 + \sqrt{c_t \cdot \varepsilon_{c1}}} \qquad \text{for } \varepsilon_{c1} > \varepsilon_t'$$
(6-23a)

and

$$\frac{1}{m} = \sum_{i=1}^{n} \frac{4\rho_{s,i}}{d_{b,i}} \cdot \left| \cos(\theta - \alpha_i) \right|$$
(6-23b)

$$c_t = 3.6 \cdot t_d \cdot m \tag{6-23c}$$

where,

 $t_d$  = directional coefficient (taken as 0.60)

 $\rho_{s,i}$  = reinforcement ratio of contribution *i*;

 $d_{b,i}$  = bar diameter of contribution *i*, (mm);

 $\theta$  = inclination of principal direction perpendicular to crack surface, (radians / degrees);

 $\alpha_i$  = inclination of reinforcement contribution *i* (radians / degrees).

When the analysis of FRC elements coupled with conventional reinforcing bars is performed, the following formulation is used (Lee et al., 2013b):

$$f_{c1} = \frac{f_t'}{1 + \sqrt{c_f \cdot 3.6 \cdot m \cdot \varepsilon_{c1}}} \qquad \text{for } \varepsilon_{c1} > \varepsilon_t'$$
(6-24a)

and

$$c_f = 0.6 + \frac{1}{0.034} \left(\frac{l_f}{d_f}\right) \frac{V_f^{1.5}}{m^{0.8}} \qquad \text{for end-hooked fibres}$$
(6-24b)

$$c_f = 0.6 + \frac{1}{0.058} \left(\frac{l_f}{d_f}\right)^{0.9} \frac{V_f}{m^{0.8}}$$
 for straight fibres (6-24c)

where *m* is as defined in Equation 6-23, and  $l_f$ ,  $d_f$ , and,  $V_f$  are characteristics of the steel fibres and are as defined in Equation 6-12.

### Crack Spacing

The addition of steel fibres can significantly influence the cracking behaviour of reinforced concrete, with fibre addition typically resulting in reduced crack spacings and reduced crack widths. Because the MCFT and DSFM behavioural models require some estimate of average

crack widths in the evaluation of the crack-slip on the local crack surface, a reliable assessment method for calculating the average crack spacing and average crack widths is pertinent.

The method used to calculate crack spacing in VecTor4 was developed by Deluce et al. (2013) and is based principally on the formulation presented by Collins and Mitchell (1997), which is an adaptation of the recommendations from the CEB-FIP 1978 Model Code. The crack spacing,  $s_{cr}$ , is directly computed in each of the principal stress directions of the concrete and, as such, is updated with each iteration performed in the VecTor4 analysis. In brief:

$$s_{cr} = 2\left(c + \frac{s_b}{10}\right)k_3 + \frac{k_1k_2}{s_m}$$
(6-25a)

where

$$s_{b} = \frac{1}{\sum_{i=1}^{n} \frac{\cos^{2} \theta_{i}}{s_{b,i}}} \le 15 \cdot d_{b,i_{\text{max}}} \qquad ; \qquad s_{b,i} = 0.5 \sqrt{\frac{\pi d_{b,i}^{2}}{\rho_{s,i}}}$$
(6-25b,c)

$$s_{m} = \sum_{i=1}^{n} \frac{\rho_{s,i}}{d_{b,i}} \cos^{2} \theta_{i} + \frac{\alpha_{f} V_{f}}{d_{f}} \cdot \max\left(\frac{l_{f} / d_{f}}{50}, 1.0\right)$$
(6-25d)

$$k_1 = 0.40;$$
  $k_2 = 0.25;$   $k_3 = 1 - \frac{\min(V_f, 0.015)}{0.015} \cdot \left[1 - \min\left(\frac{50}{l_f / d_f}, 1.0\right)\right]$  (6-25e,f,g)

In the relationships presented in Equation 6-25,

c =concrete clear cover, (mm);

 $\theta_i$  = inclination between reinforcement contribution *i*, and the considered principal direction;

 $\alpha_f$  = fibre orientation factor (taken as 0.50 for 3D infinite element) (Aveston and Kelly, 1973); all other variables are defined previously in this section of the thesis.

### Summary

The addition of steel fibres has a significant influence on many of the analysis parameters involved in the VecTor4 solution algorithm. The refinements made to the program to implement FRC analysis capabilities were developed from a relatively limited data set pertaining to SFRC, and R/FRC, elements subjected to uniaxial and biaxial loading conditions. As further data are developed, the formulations presented in this section may require additional modification.

# 6.2 Monotonic Loading: Verification

This section of the thesis presents a series of verification studies carried out to assess the performance of VecTor4 under monotonic loading conditions. Data from experimental test programs were selected such that a wide range of element types subjected to different monotonic loading conditions were considered. Because the thick-shell formulation employed in the VecTor4 analysis procedure is considered as one of the primary features which distinguishes the program from other nonlinear RC finite element analysis programs, emphasis was focused on analyses considering VecTor4's out-of-plane sectional analysis calculations.

Test data pertaining to shear-critical beams and slabs, thin-plate elements, elements subjected to combined tension and shear, elements subjected to three-dimensional loading conditions including biaxial bending and combined in-plane and out-of shear forces, and R/FRC elements subjected to in-plane shear comprise the experimental data used to verify the performance of VecTor4 under monotonic loading conditions.

# 6.2.1 VecTor4 Modelling Approach

This subsection presents a brief overview of the modelling approach used in the monotonic verification studies. General assumptions regarding material properties and selected behavioural models are discussed, and findings from a brief investigation regarding the sensitivity of the developed finite element models are presented.

# Behavioural Models and Analysis Parameters

One of the most challenging aspects in performing nonlinear finite element analyses involves the determination and selection of appropriate material and structure behavioural models. Many software programs require the user to select from a series of available behavioural models which may or may not be appropriate for the given problem at hand. The primary weakness of such an approach is that the user requires at least basic knowledge of the available models in order to confidently select those which are most appropriate for the analysis they are performing.

Because users may not have sufficient background knowledge regarding the large number of behavioural models which are typically available in most commercial software programs, the user must resort to performing supplementary research of the various available modelling options, and/or perform detailed parametric investigations to assess the impact of their model selections. Such approaches are inappropriate for practical applications, and in many instances still lead to uncertainty regarding model selection and the analytical results.

Based on these considerations, a set of 'default' material and structure behavioural models have been defined in VecTor4. The default models are based primarily on those predefined in prior studies involving other VecTor software analysis programs, and result in what is believed to be a balanced approach resulting in reasonable solution accuracy and solution stability under typical loading and analysis scenarios.

The use of default modelling options are recommended and are applicable for most analyses; however, VecTor4 does support a wide range of non-default behavioural models which may be advantageous in the analysis of certain structures possessing atypical details or loading conditions, or in cases where the user is investigating the influence of a specific behavioural mechanism. A complete listing and summarized descriptions of the available material and structure behavioural models is provided in Wong et al. (2012). The models and analysis options which have been defined as default for all VecTor software programs, and thus utilized here, are summarized below in Table 6.1.

Conci	et	e Models	<b>Reinforcement Models</b>					
Compression Base Curve	:	Hognestad (parabola) ¹	Hysteretic Response	:	Bauschinger (Seckin)			
Compression Post-Peak	:	Modified Park-Kent	Dowel Action	:	Tassios (crack slip)			
Compression Softening	:	Vecchio 1992-A						
Tension Stiffening	:	Modified Bentz	Analys	is	Options			
Tension Softening	:	Linear	Shear Analysis Mode	:	Parabolic Shear Strain			
FRC Post-Crack Tension	:	SDEM	Strain History	:	Considered			
Confined Strength	:	Kupfer / Richart	Strain Rate Effects	:	CEB/Malvar-Crawford ²			
Dilatation	:	Variable - Kupfer	Structural Damping	:	Rayleigh ²			
Cracking Criterion	:	Mohr-Coulomb (stress)	Geometric Nonlinearity	:	Considered			
Crack Stress Calculation	:	Basic (DSFM/MCFT)	Crack Allocation	:	Uniform			
Crack Width Check	:	Crack Limit (agg/2.5)	² default options for Vec	Го	r4 dynamic analysis			
Crack Slip Calculation	:	Walraven						
Hysteretic Response	:	Nonlinear w/ Plastic Offsets						

Table 6.1 – VecTor4 Default Behavioural Models

¹ Popovics HSC model for high strength concretes (see below)

In the analyses used to verify the performance of VecTor4, the default models and analysis options presented above have been used exclusively with the exception of the concrete

compression base curve. In the case of high strength concretes (HSC), the pre-peak stress-strain response tends to be much more linear than that of normal strength concretes (NSC). Because of this, the parabolic compression base curve model which is the default option has the tendency to overestimate the pre-peak stiffnesses of HSC. As such, for the case of NSC, Hognestad's parabola was considered in the analyses, and the model referred to as Popovics HSC was used for concretes with compressive strengths greater than 45 MPa. Note that Popovics HSC model refers to that presented by Collins and Porasz (1989), and is based principally on the works of Thorenfeldt et al. (1987) and Popovics (1973).

In all of the analyses performed in this study, the concrete tensile strength,  $f'_t$ , has been estimated using the following relationship:

$$f'_{t} = 0.33 \cdot \sqrt{f'_{c}} \tag{6-26}$$

The use of Equation 6-26 to calculate the direct tensile strength has been used, and is recommended, for several reasons: the direct tensile strength of concrete is difficult to obtain experimentally and as a result is often not provided in the literature, the tensile strengths determined from material tests of plain concrete are not necessarily indicative of actual tensile strengths of reinforced concrete, and the DSFM and MCFT were developed using the above relationship as the basis of estimating concrete tensile strength.

Lastly, all analyses have been performed using under-relaxation with a constant averaging factor of 0.50 and a specified convergence tolerance of 1.000010. By default, convergence is calculated on the basis of nodal displacements (translations and rotations), and the local strain values used to evaluate the material matrix are averaged from one iteration to the next.

### Element Density and Layer Discretization

To investigate the sensitivity of user developed meshing parameters, such as element size and the number of concrete layers used to represent the cross section, a brief parametric study was performed. Three reinforced concrete elements were selected for investigation: i) a thin-plate under combined in-plane compression and out-of-plane loading, (ii) a shear-critical beam which contained out-of-plane shear reinforcement, and (iii) a shear-critical beam with no out-of-plane

reinforcement. The selected elements represent test specimens from experimental programs available in literature. Details of the test programs involving the selected specimens are summarized in the following sections of this chapter.

The thin-plate specimen under consideration is a flexure-critical element which failed experimentally due to geometric instability, arising from the combined in-plane normal compression forces and out-of-plane surface pressure. A series of five different meshes were used to assess the influence of element density on the computed load-deflection response of the plate (see Figure 6.7). To easily accommodate different element sizes, discrete loading locations corresponding to the out-of-plane surface pressure were simplified and represented as uniformly distributed load acting on the plate's surface.



Figure 6.7 – Thin-Plate Mesh Configurations (dimensions in millimetres)

From Figure 6.8a, it is evident that the computed ultimate strength and ductility of the plate were only marginally affected by mesh density and the number of concrete layers considered in the analyses. With the exception of the analysis performed using the coarsest mesh considered ( $2 \times 2$  grid of elements used to represent one-quarter of the plate), less than 5 % variation in the

ultimate strength of the plate was calculated as a result of decreasing element size. Higher levels of variation were determined with respect to the plate's computed ductility; however, other factors such as the selected convergence criteria, load-step size, and the selected averaging factor can also influence the computed deformations at the ultimate capacity when performing load-controlled analyses.



Figure 6.8 – Mesh Sensitivity; Thin-Plate Specimen A1

To investigate the influence of layer discretization on the analysis of the thin-plate, the number of layers used to represent the cross section was varied from ten to fifty equal thickness concrete layers. From Figure 6.8b, it can be seen that little benefit resulted from the use of more than fifteen concrete layers. In all cases, the computed strength and stiffness of the plate was identical. With only ten layers used to represent the concrete cross section, only minor variation of the ultimate deformation was calculated.

In the case of shear-critical beam VS-A1, it was found that some level of mesh dependency was present, particularly with respect to the computed ductility of the beam (see Figure 6.9). Meshes with typical element sizes ranging from approximately 20 % to 85 % of the overall height of the beam were considered; however, it should be noted that three of the four meshes considered used a constant element size to represent the region of increased shear reinforcement located in the midspan of the beam (see Figure 6.10). Note that details regarding the test beam are provided in

Section 6.2.2. Analytically, failure of the beam was found to occur within the region of increased shear reinforcement after yielding of the longitudinal reinforcement had occurred. As such, it was determined that variation of the typical element size had little effect on the computed analytical response. Subdividing the shell element comprising the failure region of the beam had no effect on the calculated strength and stiffness of the beam; however, it did result in a significant decrease in computed ductility.



Figure 6.9 – Mesh Sensitivity; Beam VS-A1



Figure 6.10 – Beam VS-A1 Mesh Configurations (dimensions in millimetres)

As discussed previously, no form of shear strength enhancement is considered in VecTor4 by default. Localized out-of-plane confining stresses which occur within disturbed regions are neglected and, as such, members critical with respect to out-of-plane shear which are located

within disturbed regions may display some form of mesh dependency. The computed response variation attributed to mesh density effects in VecTor4 seems to be primarily limited to the computed ductility. VecTor4 calculations pertaining to strength and stiffness characteristics are much less sensitive to the element sizing.

A similar set of analyses performed for Beam VS-OA1 supports the general findings stated above. Note that VS-OA1 is nominally identical to VS-A1; however, it does not contain shear reinforcement. Unlike the computed responses obtained for the moderately ductile shear-critical beam, varying the element size was shown to have negligible effects on the computed response for a beam exhibiting a non-ductile failure (see Figure 6.11). Elements ranging from approximately 11 % to 55 % of the height were considered in the analysis of VS-OA1, and the impact on the computed strength and stiffness was found to be negligible. Given that no significant levels of ductility were achieved with any of the meshes considered, the mesh sensitivity in this case was insignificant.



Figure 6.11 – Mesh Sensitivity; Beam VS-OA1

Lastly, for the shear-critical beams considered, the influence of the number of layers used to represent the beams' cross section was also investigated. Based on the figures above, and in agreement with that observed from the analyses of the thin-plate element, the number of concrete layers used to represent the cross section had little effect on the computed load-deflection responses of the beams. However, not apparent from the above figures, the beam analyses

performed using more concrete layers (thirty to forty-five layers) did tend to achieve better convergence than those which considered fewer layers. Additionally, in the case of Beam VS-A1, analyses performed using fewer layers resulted in less stable computed responses, particularly after the onset of longitudinal steel yielding.

Based upon the results from the parametric investigation, it is evident that mesh density does have some level of influence on the VecTor4 results. In all cases considered, it was found that varying the size of the shell elements used to construct the finite element meshes had marginal effects on the calculated strengths and stiffnesses of the members. The computed ductility, however, was found to be significantly more sensitive to element sizing, particularly with respect to ductile members which are subjected to significant out-of-plane shear forces.

For typical elements subjected to out-of-plane shear or combined membrane forces and out-ofplane shear, the following approach was adopted in creating the finite element meshes used in the reported verification studies. The guidelines were selected such that the resulting mesh densities were fine enough to accommodate variations in member geometry, loading and boundary conditions, and stiffness degradation throughout the structures, while still being capable of providing reasonable estimates of structure ductility:

- i) the planar geometries of the shell elements were limited to a minimum dimension of approximately 40 % of the total height of the element;
- ii) the shear span was modelled using a minimum of 5 to 6 shell elements;
- iii) the concrete section was subdivided into 25 to 30 concrete layers.

In many cases the above provisions regarding element sizing (recommendations i and ii) cannot both be satisfied; this is particularly true as the span-to-depth ratio of the member or structure decreases. However, typical shell structures (e.g., tanks, silos, thin-shells/plates) possess relatively low to moderate out-of-plane thicknesses in comparison to their in-plane geometries, and will generally satisfy the suggested approach.

In structures where loads are predominantly resisted by way of membrane loads or bending, the above modelling approach is less suitable. For example, the thin-plate element considered in the mesh density study was governed primarily by flexural failure mechanisms, and it was

determined that reasonable estimates of the plate's capacity and deformation at peak loadresistance were attained with as few as 3 shell elements (in each planar direction) comprising the shear span, with planar dimensions being approximately 4.5h in each direction. Additionally, it was determined that as few as 15 concrete layers provided a reasonable representation of the element cross section.

Therefore, because a wide array of structures possessing variable geometries, load, and restraint conditions can be modelled using VecTor4, it is not possible to provide a single set of modelling guidelines which can be adopted for all scenarios. As such, it is recommended that some level of mesh density verification be performed when using VecTor4, particularly when estimates of ductility and deformation are of principal interest.

# **6.2.2 Shear-Critical Beams**

A test series performed by Bresler and Scordelis (1963), focusing on the behaviour of shearcritical reinforced concrete beams, is frequently used as a benchmark for corroborating finite element programs and analytical procedures. Its frequent use for verification purposes has been attributed to the detailed documentation of the testing program, and to the fact that the behaviours exhibited by the beams in the program have proven to be a difficult challenge to simulate using analytical methods.

In 2004, an experimental program was performed at the University of Toronto (Vecchio and Shim, 2004) in an effort to recreate, as much as possible, the classic Bresler-Scordelis test series performed in the 1960s. The primary goals of the program were to assess the repeatability of the results obtained from the Bresler-Scordelis tests, and to gather additional data regarding the post-peak behaviour of the shear-critical beams, data not reported from the original tests. In all, twelve beams were tested to failure under three-point monotonic loading. The beams were constructed such that they were nominally identical to the Bresler-Scordelis beams in terms of geometry and reinforcement level. Some deviation from the original test series with respect to material strengths, particularly for the concrete used in the Toronto beams, was encountered. The cross section details of the Vecchio-Shim (VS) beams are presented in Figure 6.12.



Figure 6.12 – Cross Section Details for VS Beams (dimensions in millimetres)

The beams were divided into three series of tests (Series 1, 2, and 3), with four beams comprising each series (Beam OA, A, B, and C). Series 1 beams had a clear span of 3.66 m; beams within Series 2 had a clear span of 4.57 m; and the beams forming Series 3 had a clear span of 6.40 m (see Figure 6.13). All beams had a cross-sectional depth of 552 mm, and were tested under simple support conditions. Load was incrementally applied at the midspan in a displacement-controlled manner. Resulting load-deflection behaviour, crack widths and patterns, and steel strains were documented throughout testing.

The material properties of the reinforcement and the concrete, obtained from tensile coupon and compressive cylinder tests, are presented in Table 6.2.



Figure 6.13 – Elevation Details for VS Beams (dimensions in millimetres)

		Reinforcement					
	Diameter	Area	$f_{v}$	$f_u$	$E_s$	$\mathcal{E}_{\mathrm{sh}}$	$\mathcal{E}_{\mathrm{u}}$
Bar Size	(mm)	$(mm^2)$	(MPa)	(MPa)	(MPa)	$(x \ 10^{-3})$	$(x \ 10^{-3})$
M10	11.3	100	315	460	200,000	7.7	207
M25 ^a	25.2	500	440	615	210,000	7.5	200
M25 ^b	25.2	500	445	680	220,000	8.5	216
M30	29.9	700	436	700	200,000	11.4	175
D4	5.7	25.7	600	651	200,000	3.0	38
D5	6.4	32.2	600	649	200,000	3.0	35
				Concre	ete		
		$f_c'$	$\mathcal{E}_0$		$E_c$	$E_{c}$	
Beam Se	ries	(MPa)	(x	10 ⁻³ )	(MPa)		(MPa)
Series	1	22.6	1	.60	36,500		2.37
Series 2		25.9	2	2.10			3.37
Series	3	43.5	1	.90	34,300		3.13

	Table 6.2 -	- Material	Properties	of VS	Beams
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¹ maximum aggregate size = 20 mm

^a Series 2

^b Series 1 and 3

Taking advantage of the symmetry of the beams and the test setup, a quarter-beam model was developed for each of the twelve specimens. Note that because the central nodes of the shell elements do not consider translational dofs, quarter-beam models require less computation than half-beam models constructed with the same number of elements. The element sizing was selected such that the stiffness degradation along the beam length would be sufficiently captured, and variations in provided transverse reinforcement could be accommodated. Series 1 beams

were modelled using eight shell elements with a typical length of 238 mm; Series 2 beams were modelled using ten shell elements with typical length of 236 mm, and the Series 3 beams were modelled using shell thirteen elements, with a typical element length of 254 mm. Load was applied in a displacement controlled manner, to nodes located at the midspan of the beams (nodes 17, 34, and 51 for Series 1 beams). The finite element mesh developed, and support restraints provided (denoted by 'R'), for Beam VS-A1 are illustrated in Figure 6.14.



Figure 6.14 – Mesh Developed for Series 1 Beams (dimensions in millimetres)

For all series of beams, the cross section was divided into thirty concrete layers. Longitudinal reinforcement was modelled using discrete steel layers located within the depth of the section. The transverse shear reinforcement was treated as a property of the concrete layers. Horizontal reinforcement components provided by the closed stirrups were considered and modelled discretely over the height of the beam. It should be noted that the inclusion of the horizontal reinforcement contribution from the closed stirrups can often result in significant local strength and ductility enhancements. The sectional model details for beam VS-A1 have been presented in Figure 6.15.



Figure 6.15 – Sectional Model; Typical Element for Beam VS-A1 (dimensions in millimetres)

Displacement increments of 0.25 mm, 0.50 mm, and 1.00 mm were used for the beams containing shear reinforcement within Series 1, 2, and 3, respectively. The analytical load-deflection behaviour for the full test series has been plotted alongside experimental results, and is presented in Figure 5. A comparison of the peak load values and corresponding midspan displacements are summarized in Table 6.3.

Based upon the results presented in Table 6.3 and in Figure 6.16, it can be seen that VecTor4 was capable of computing the capacities of the beams with a high level of accuracy, resulting in a mean analytical to experimental ratio of 1.01, with a coefficient of variation of 6.0 %. The least accurate load estimates were obtained for Beams VS-OA2 and VS-C2, with analytical-to-experimental ratios of 1.13 and 1.12, respectively.

		Peak Load	(kN)	Displac	Displacement at Peak Load (mm)		
Beam	Test	VecTor4	VecTor4/Test	Test	VecTor4	VecTor4/Test	
OA1	331	348	1.05	9.1	8.8	0.96	
A1	459	438	0.96	18.8	23.3	1.24	
B1	434	433	1.00	22.0	21.5	0.98	
C1	282	265	0.94	21.0	19.5	0.93	
OA2	320	360	1.13	13.2	13.0	0.98	
A2	439	446	1.02	29.1	23.0	0.79	
B2	365	354	0.97	31.6	28.5	0.90	
C2	290	325	1.12	25.7	25.0	0.97	
OA3	385	406	1.05	32.4	33.0	1.02	
A3	420	406	0.97	51.0	32.0	0.63	
B3	342	334	0.98	59.6	46.0	0.77	
C3	265	256	0.97	44.3	42.0	0.95	
		Mean =	1.01		Mean =	0.93	
		C.O.V. =	6.0 %		C.O.V. =	15.5 %	

Table 6.3 – Summary of Results for VS Beams

In the case of Beam VS-OA2, which did not contain shear reinforcement, the 13 % overestimation of capacity is deemed reasonable since the shear capacity of the beam will be highly sensitive to the approximated concrete tensile strength. For Beam VS-C2, the ultimate capacity of the beam, both experimentally and analytically, was controlled by a shear-compression failure







within the midspan region. As such, the analytical failure load of the beam will be sensitive to assumptions made regarding the concrete compression constitutive models, inclusion of confinement effects and lateral expansions, and the assumed post-peak compressive response.

VecTor4 predicted the displacements at peak load well with a mean analytical-to-experimental value of 0.94, and a coefficient of variation value of 15.5 %. This value is deemed reasonable for displacement calculations as typically a larger level of variance is encountered when calculating member displacement behaviour. In terms of post-peak response, VecTor4 tended to underestimate the ductilities of the beams.

## 6.2.3 Plates under Combined In-Plane and Out-of-Plane Loads

An experimental program carried out at the University of Alberta (Aghayere and MacGregor, 1990) was performed to assess the behaviour of lightly reinforced, thin concrete plates subjected to combined axial compression and out-of-plane shear. Results from the test program comprise an interesting data set in that many of the specimens exhibited instability or buckling type failure modes as opposed to material governed failures. Given that shell structures typically rely heavily on resisting loads through a combination of in-plane and out-of-plane loads, this test series serves as a practical verification study, and additionally serves as means of verifying the geometric nonlinearity algorithm in VecTor4.

The test program was comprised of nine plate elements which were constructed with varied geometries, aspect ratios, and reinforcement levels, and were subjected to differing levels of axial compression. The plates forming the test program were subdivided into four series of tests: Series A, B, C and D. The results from Series A and B were presented in detail in a companion paper (Massicotte et al., 1990), and as such, only the five tests comprising these series have been selected for verification.

Series A plates were 1830 mm square, while the two plates forming Series B were rectangular with dimensions of 1830 mm x 2745 mm. A summary of the key material properties and loading conditions for the plates forming the two series of tests is presented in Table 6.4.

Plates which were subjected to combined in-plane and out-of-plane loading were loaded with axial compression in the *y*-direction (see Figure 6.17). In the case of the B-series rectangular plates, axial compression was applied to the shorter dimension. The transverse loading was applied on a 610 mm grid, resulting in nine load application points for the A-series plates, and twelve loading points for the B-series plates. The bases of the plates were supported by a series of rollers. The number of rollers used varied from twelve to sixteen depending on the geometry of the specimen. Lastly, to represent a true simply-supported condition, the corners of the plates were restrained vertically to prevent uplift from occurring.

Plate	Ny (kN/m)	t (mm)	$l_{n,x}$ (mm)	$l_{n,y}$ (mm)	l _{n,x} / l _{n,y}	<i>f</i> _c ^{, 3} (MPa)	E _{cs} (MPa)	$\rho_x^{1,2}$ (%)	${ ho_y}^{1,2}$ (%)
A1	962	67.0	1830	1830	1.0	32.3	22,970	0.336	0.390
A2	765	64.2	1830	1830	1.0	32.3	23,010	0.350	0.400
A3	0	65.3	1830	1830	1.0	32.2	23,150	0.344	0.400
B1	874	64.5	1830	2745	1.5	40.3	25,580	0.500	0.590
B2	634	64.8	1830	2745	1.5	40.2	25,550	0.500	0.590

Table 6.4 – Properties of Alberta Plates

¹ per layer of steel (each slab contained two layers of steel in each orthogonal direction)

² comprised of  $\phi 6.35$  mm bars ( $f_y = 504$  MPa;  $E_s = 197,300$  MPa;  $f_{su} = 640$  MPa;  $\varepsilon_u = 45.5 \times 10^{-3}$  mm/mm)

³ maximum aggregate size = 14 mm

In all cases, one-quarter of the plate was modelled in VecTor4. From the mesh density investigation performed in Section 6.2.1, it was determined that sixteen shell elements (a 4 x 4 grid) were sufficient in providing an accurate representation of the A-series plate specimens. However, to accommodate the locations of the discrete loading points used in the testing program, the element geometry required modification. Twenty-five shell elements with dimensions ranging from 153 mm to 203 mm square were considered for the A-series plates, and a total of thirty-five shell elements were used to model the B-series plates. To simplify the geometry of the model, continuous vertical restraint was provided along the outside of the plate.

Nodes corresponding to the plate centerlines were restrained longitudinally and rotationally to enforce symmetry. When present, axial compression loads were applied along the free edge perpendicular to the y-direction. The out-of-plane loads were applied discretely, at locations corresponding to those used in the experimental program. An illustration of the mesh used for Plate A1 is provided in Figure 6.18.



Figure 6.17 – Alberta Plate Tests; Series A Plate (dimensions in millimetres)

Because the plates in this study were relatively thin, the shell elements were subdivided into fifteen concrete layers, resulting in a concrete layer thickness of approximately 4 mm. An additional four layers were used to represent the longitudinal layers of steel. Axial compressive loads were modelled as a series of constant axial forces, and the out-of-plane load was applied using a load step size of 0.5 kN applied to the quarter-plate specimen.



Figure 6.18 – Mesh Used for Plate A1 (dimensions in millimetres)
The analytical load versus deflection responses have been plotted alongside the experimental results in Figure 6.19. VecTor4 was effective in capturing the initial stiffnesses, the exhibited softening behaviours, and the ductilities of the plates. It is interesting to note that the analyses performed without geometric nonlinearity resulted in significant overestimation of the capacities of the plates subjected to combined compression and shear. The least accurate analytical response was obtained for specimen A3 which was subjected to only out-of-plane shear; in this case the VecTor4 analysis underestimated the strength and ductility of the plate, resulting in an analytical-to-experimental strength ratio of 0.87.

# 6.2.4 Combined Tension and Shear

Of particular interest in assessing VecTor4's applicability in modelling reinforced concrete shell structures lies in its effectiveness in capturing the behaviour of RC members under combined tension and out-of-plane shear. Many RC shell structures such as storage containers, tanks, and the silo structure example presented in Chapter 2 rely heavily on resisting internal storage loads through the development of axial tension forces in the structure wall. The occurrence of non-uniform loading conditions or varied geometry can lead to the development of large out-of-plane shear forces in the walls of these structures which often contain little or no out-of-plane (through thickness) shear reinforcement.

Three experimental studies performed on simple one-way beam and slab members without shear reinforcement have been considered for verification. A brief description of the modelling approach and the resulting VecTor4 strength calculations has been provided for each test series.

# Mattock Beams

In the late 1960s a series of reinforced concrete beams was tested by Mattock (1969) in an effort to evaluate the influence of axial loading on the shear strength of members without shear reinforcement. The beams were simply supported and tested under three-point midspan loading conditions, with axial forces applied at the support locations. All of the beams were rectangular with uniform cross section geometries (see Figure 6.20). The program considered several variables: applied axial force, provided longitudinal reinforcement ratio, the concrete compressive strength, and the effective shear span.



Figure 6.19 - Load versus Midpoint Deflection Behaviours for Alberta Plates



Figure 6.20 – Details of Mattock Beams (dimensions in millimetres)

In total, thirty-one RC beams with varied material compositions and varied axial loading conditions were tested. Eleven of the beams were tested under combined axial tension and shear. Although the primary goal of this section is to assess VecTor4's ability to estimate the capacity of elements under combined tension and shear, all thirty-one beams have been modelled, and the resulting capacity calculations are presented herein.

The mesh used to create the finite element model for the small-span beams (Beams M1 through M14), is presented in Figure 6.21 (note that 'R' represents a support restraint, and the shaded areas enclose nodes with applied forces denoted by 'F'). Taking advantage of symmetry, quarter-section models of the beams were considered. Small-span beams were modelled using six shell elements with a typical element length of 127 mm, and the longer beams were modelled using nine shell elements with a typical element length of 152 mm. When present, axial load was applied to the longitudinal roller supports provided at the beam ends and shear was applied at the midspan nodes.



Figure 6.21 – Mesh for Mattock Short-Span Beams (dimensions in millimetres)

The beams were subdivided into twenty-five concrete layers of 12 mm thickness, and either one or two steel layers were used to represent the longitudinal bars, depending on the longitudinal reinforcement ratio. The steel reinforcing bars were reported to have yield strengths of 400 MPa, and were assumed to have a Young's modulus of 200,000 MPa. Axial loads were modelled as constant axial forces, and the shear force was applied using a load step size of 1 kN. The resulting strength estimates from the analyses of the Mattock beams are presented in Figure 6.22.



Figure 6.22 – VecTor4 Capacity Calculations for Mattock Beams

From the above figure, it can be seen that the capacity of most of the beams were computed within a reasonable level of accuracy considering that beams without shear reinforcement typically exhibit a somewhat higher level of scatter in comparison to beams with shear reinforcement. It is interesting to note that the largest over-predictions were encountered for beams with applied axial compression or no axial force applied. The capacities of the beams with applied axial tension were, on average, estimated with the highest accuracy.

The results from the experimental program and the VecTor4 analyses are tabulated in Table 6.5 and Table 6.6. For beams loaded under combined axial tension and shear, the VecTor4 analyses resulted in a mean analytical-to-experimental capacity ratio of 1.04, with a coefficient of

variation of 9.9 %. When the results from all thirty-one beams are considered, the mean analytical-to-experimental capacity ratio increased to a value of 1.06, with a coefficient of variation of 11 %.

Doom	a/d	$f_c$ '	$ ho_{long}$	N/A	V _u Exp	V _u VT4	VT4 /
Dealli	u/u	(MPa)	(%)	(MPa)	(kN)	(kN)	Exp
M1	2.7	17.1	1.03	0	36.5	42.8	1.17
M2	2.7	15.5	1.03	1.31 C	40.9	44.8	1.09
M3	2.7	46.9	1.03	0	54.7	51.0	0.93
M4	2.7	46.2	1.03	0.62 T	44.5	46.5	1.05
M5	2.7	16.1	2.07	0.62 T	33.4	40.0	1.20
M6	2.7	18.3	2.07	1.31 C	53.8	59.0	1.10
M7	2.7	17.4	2.07	2.76 C	60.1	58.9	0.98
M8	2.7	43.7	2.07	1.31 C	92.5	87.5	0.95
M9	2.7	48.0	2.07	2.76 C	105	96.7	0.92
M10	2.7	18.5	3.10	0	56.0	58.0	1.03
M11	2.7	15.2	3.10	1.31 T	42.3	45.5	1.08
M12	2.7	16.2	3.10	1.31 C	61.8	62.2	1.01
M13	2.7	49.8	3.10	1.31 C	89.0	103	1.16
M14	2.7	49.0	3.10	2.76 C	111	109	0.98
M15	5.1	25.9	3.10	0	31.1	27.2	0.87
M16	5.1	30.3	3.10	1.03 T	28.0	25.5	0.91
M17	5.1	27.8	3.10	2.76 C	37.8	33.7	0.89
M18	5.1	18.1	2.07	0	35.6	42.0	1.18
M19	5.1	18.5	2.07	0.62 T	40.1	46.0	1.15
M20	5.1	48.3	2.07	0.62 T	57.8	53.0	0.92
M21	5.1	50.5	2.07	1.31 T	56.9	51.0	0.90
M22	5.1	16.1	3.10	0	40.0	47.2	1.18
M23	5.1	18.5	3.10	0.62 T	42.3	47.0	1.11
* M24	5.1	29.2	3.10	0	52.5	67.0	1.28
M25	5.1	27.6	3.10	1.03 T	51.2	53.0	1.04
M26	5.1	28.8	3.10	1.72 T	42.3	42.0	0.99
* M27	5.1	30.5	3.10	1.38 C	53.4	66.0	1.24
M28	5.1	24.3	3.10	2.76 C	52.5	57.6	1.10
M29	5.1	53.2	3.10	0.62 T	66.7	75.0	1.12
M30	5.1	55.2	3.10	1.31 C	66.7	79.0	1.18
* M31	5.1	52.7	3.10	2.76 C	66.7	81.0	1.21

Table 6.5 – VecTor4 Capacity Calculations for Mattock Beams

C = axial compression

T = axial tension

* largest VecTor4 over-predictions

Table 0.0 - Summary of Manock Deams Successing Calculation	able 6.6 – Summar	v of Mattock Beams	Strength	Calculation
------------------------------------------------------------	-------------------	--------------------	----------	-------------

Axial Force	# specimens	Mean (VT4 / Exp)	C.O.V. (%)
tension	11	1.04	9.9
compression	13	1.06	10.9
none	7	1.09	13.6
all beams	31	1.06	11.0

# Leonhardt Slab Strips

An experimental program performed by Leonhardt et al. (1977) was carried out to assess the effects of restraint-induced axial force which commonly occurs in statically indeterminate RC structures as a result of thermal and/or shrinkage straining. The test program consisted of two full-scale slab strips subjected to combined axial tension and shear, and a series of T-beams with shear reinforcement which were pre-cracked under axial tension and then loaded in transverse shear. For the purpose of verifying VecTor4, only the two slab strip specimens which did not contain shear reinforcement have been considered.

The slab specimens were tested under four-point skew symmetric loading conditions to represent a portion of a continuous slab at the location of contraflexure, where the shear forces are high and the applied moment is low. The slab strips were 275 mm thick, 760 mm wide, and 5 metres in length. With the exception of the end regions where axial tension was applied, the slabs contained no shear reinforcement (see Figure 6.23). At the midspan of the slab, the inner layers of longitudinal reinforcement were terminated with no overlap. Lastly, in an effort to force the slab to initially crack at specific, heavily instrumented locations under the applied axial tension, the slabs were notched at three locations along their lengths: i) 625 mm from the cantilever loading point, ii) 500 mm from the end support, and iii) at midspan. The notches were created during the casting process and were included on all surfaces (top, bottom, and side faces). Key properties of the two slab strips, identified as M5 and M6, are summarized in Table 6.7. Note that the failure loads,  $P_u$ , of the two slabs which were reported to be equal, are also included in the table.



Figure 6.23 – Loading Conditions for Leonhardt Slabs (adapted from Leonhardt et al., 1977)

Due to the skew symmetric loading conditions used in the tests, a finite element mesh which represented the full length of the slabs and half of the width was constructed using sixteen shell

elements. To simplify the model, the axial tension force was applied at the same location as the transverse force acting on the slab cantilever. Additionally, overhanging segments beyond the cantilever load and end support were not modelled. The mesh developed to model the slabs is presented in Figure 6.24 (note that 'R' represents a support restraint, and the shaded areas enclose nodes with applied forces denoted by 'F'). To accommodate the three notched regions of the slabs, the element geometry was adjusted at discrete nodal points corresponding to the notch locations. Similarly, the thickness of the elements was also reduced at the locations of the fabricated notches.

Table 6.7 – Key Properties and Reported Failure Loads

Slab	N/A (MPa)	<i>fc</i> ^{'1} (MPa)	$ ho_{long} \ (\%)$	fy (MPa)	$P_u$ (kN)
M5	1.40	25.8	0.73	600	137
M6	1.90	21.1	0.91	640	137

¹ maximum aggregate size = 20 mm





In the analysis, the applied axial tension force was held constant and the transverse loading was applied as a series of point loads corresponding to a load stage increment of P = 3 kN.

The analyses resulted in ultimate failure loads of 135 kN and 126 kN, corresponding to slabs M5 and M6, respectively. These values agree well with the ultimate failure load of 137 kN reported in the experimental work. In both slabs, a shear failure mode was estimated to occur at the notched region of the slabs' cantilever. This agrees with the reported failure mode for slab M6; however, it differs from the midspan shear failure reported for slab M5. This deviation from the experimental result is most likely due to the fact that no special consideration was given to the bar cut-offs at the midspan location, and the fact that the sectional analysis considered in VecTor4 does not accommodate diagonal crack propagation. As such, the observed shear failure

propagated by a large diagonal crack at the midspan which did not engage the inner layers of reinforcement was not accurately captured in the finite element model.



Figure 6.25 – Shear Failure of Slab M5 (adapted from Leonhardt et al., 1977)

Nevertheless, the resulting failure loads and overall deformation behaviours of the slabs were estimated well. In Figure 6.26, the deformed shapes of the slabs have been compared with results from the VecTor4 analyses at two load levels:  $P = 0.6 \cdot P_{u,exp}$ , and  $P = 0.9 \cdot P_{u,exp}$ . Reasonable displacement estimates were obtained for both slabs, at both load levels.



Figure 6.26 - VecTor4 Slab Deformation Results

#### COSMAR Beams

In the late 1970s and early 1980s, a series of investigations were undertaken as part of a collaborative research effort to address, and to further advance, the current state of knowledge regarding the consideration of shear in the analysis and design of reinforced concrete offshore structures. The joint project was performed as part of an initiative set forth by *COSMAR*: Concrete Structures for Marine Production, Storage, and Transportation of Hydrocarbons.

As one of the research groups contributing to the static strength investigations, Sørenson et al. (1981) performed a series of tests on reinforced concrete beams subjected to combined axial tension and shear forces. The aim of the study was to further contribute to what was, and still is, a limited database of experimental results pertaining to reinforced concrete members under combined tension and shear, and to compare with the limits prescribed by governing codes and standards available at that time.

The testing program consisted of a series of reinforced concrete beams which contained relatively large quantities of symmetrically placed longitudinal reinforcement, but contained no shear reinforcement. The beams were tested using a four-point bending scheme similar to that of the Leonhardt et al. slab strips (see Figure 6.27). The concrete strength of the beams was, on average, about 53 MPa with a maximum aggregate size of 16 mm. The majority of the beams contained 1.80 % longitudinal reinforcement in both the positive and negative bending regions. The program spanned a number of testing variables; however, the main emphasis was focused on the applied axial tension which varied from zero to approximately 7.3 MPa (440 kN).



Figure 6.27 – Details of Typical COSMAR Beam (dimensions in millimetres)

The constructed finite element mesh consisted of twelve shell elements with an approximate longitudinal length of 150 mm, with minor geometric deviations to accommodate locations of the applied loads and support restraints. As was done in the case of the Leonhardt slabs, the model was simplified such that axial tensile forces were applied at the same location as the transverse force acting on the cantilever portion of the beam. Additionally, segments of the beam extending beyond the outermost loads and supports were neglected. The internal vertical support restraint was treated as a longitudinal roller and the outside vertical restraint was modelled as a pin. The mesh developed for the *COSMAR* beams is presented in Figure 6.28. Because of the non-

symmetric loading scheme, the full length of the beam was considered, but only half of the beam width.



Figure 6.28 – Mesh for Typical COSMAR Beams

The axial tension was applied at the beginning of the analyses and held constant throughout. The shear force was applied in fixed load increments equivalent to P2 = 1.5 kN.

The capacities of the *COSMAR* beams were under-estimated to varying degrees. From Figure 6.29, it can be seen that the computed shear strength-tensile force failure envelope from the VecTor4 analyses estimated the member capacity reasonably well at higher tensile force levels, but is quite conservative at the lower tensile force levels. It is also interesting to note that beam specimens T2, T3, and T14 were nominally identical, yet their resulting experimental shear capacities varied by as much as 40 %. Previous analytical investigations employing the MCFT to analyze the *COSMAR* beams have resulted in similar levels of conservative strength estimates (Bhide and Collins, 1989).



Figure 6.29 - VecTor4 Shear Strength Computations for COSMAR Beams

# 6.2.5 Reinforced Concrete Shells and Slabs

In assessing the performance of VecTor4, it is important that experimental data from test programs involving reinforced concrete elements subjected to combined in-plane and out-of-plane loads and three-dimensional loading conditions be considered. The ability to analyze such elements is what distinguishes VecTor4 from two-dimensional analysis programs, and from other shell formulations which neglect out-of-plane shear.

Two experimental programs involving reinforced concrete shell elements, both of which were performed using the University of Toronto's Shell Element Tester (SET), have been considered for verification. A general overview of the SET frame is provided by Kirschner and Collins (1986). An experimental program consisting of large-scale one-way reinforced concrete slab specimens has also been considered. The slabs form an interesting data set in that the observed behaviours and failure modes of the specimens were strongly influenced by the development of three-dimensional stress conditions.

# Adebar Shells

In the late 1980s an experimental program performed by Adebar (1989) involved the testing of nine reinforced concrete shell elements. One of the primary goals of the program was to investigate the interaction behaviour of reinforced concrete subjected to combined in-plane and out-plane shear. The acquired experimental data were used to corroborate a shell analysis program which was also developed as part of the research program (Adebar and Collins, 1991).

The shell specimens were 1625 mm square with thicknesses ranging from 310 mm to 410 mm. They were reinforced with two sets of orthogonal reinforcement grids, resulting in four layers of longitudinal reinforcement per specimen. The orientation of the reinforcement varied amongst the shell specimens, with some specimens having reinforcement aligned in the directions of the x and y axes, and others having reinforcement skewed 45° relative to the x and y axes (referred to as the x'-y' axes). All of the shell specimens possessed equal amounts of reinforcement in the orthogonal directions (x-y or x'-y' directions). Small amounts of transverse reinforcement were also provided in the form of single-leg stirrups or 'T-headed' reinforcing bars. An illustration of the orthogonal reinforcement grid pertaining to one of the shell elements with skewed reinforcement is provided in Figure 6.30a, and an illustration of the typical cross section for the

shells in this test program is provided in Figure 6.30b. A summary of the key material properties and the loading conditions pertaining to five selected shell elements is provided in Table 6.8.





Specimen	Applied	thickness	$f_c$	max agg	$f_y$	$\rho_{x'} = \rho_{y'} *$	$\rho_z$
-	Loading	(mm)	(MPa)	(mm)	(MPa)	(%)	(%)
SP3	[†] ○ ● ●	310	49.8	10	480	3.58	0.08
SP4	$\rightarrow \uparrow \circ \bigcirc \circ \circ$	310	52.4	10	480	3.58	0.08
SP7	$ \begin{array}{c} \downarrow 4V \\ \bullet \uparrow \circ & \bigcirc \circ \uparrow \\ \uparrow & \circ & \circ & \circ \\ \downarrow & \circ & \circ & \circ \\ \uparrow & \circ & \circ & \circ \\ \uparrow & \circ & \circ & \circ \\ \uparrow & \circ & \circ & \circ \\ \downarrow & \circ & \circ & \circ \\ \bullet & \circ & \circ & \circ \\ \circ & \circ & \circ & \circ \\ \circ & \circ & \circ$	310	54.1	20	536	3.75	0.08
SP8	$\begin{array}{c} \downarrow \\ \leftarrow & & \\ \uparrow & \\ \uparrow & \\ \end{array} \rightarrow$	310	52.9	20	536	3.75	0.08
SP9	$ \begin{array}{c} \downarrow BV \\ \leftarrow \uparrow \circ \overbrace{\downarrow} BV \\ \bigcirc & \downarrow \\ 0 \\ \uparrow \end{array} $	310	49.6	20	536	3.75	0.08

Table 6.8 - Summary of Selected Adebar Shell Specimens

* total steel area in each direction

Because the shell specimens were subjected to out-of-plane shear and bending moments about a single axis (*y*-axis), a finite element mesh representing half of the planar area of the shells was considered (see Figure 6.31). Although the in-plane and out-of-plane shearing forces were applied such that they were uniform along the edges of the shell element, variation of the bending moment over the length of the specimens required that multiple elements be used to ensure that stiffness degradation throughout the specimen was sufficiently captured. Eighteen elements with dimensions of 271 mm square were used to develop the half-specimen model. The shells were restrained such that longitudinal rollers were provided along the centerlines of the specimens, and the ends of the specimens were rotationally restrained. This restraint method deviates somewhat from that used in the experimental program; however, it simplified the application of the proportional loading schemes used in the tests, and resulted in the same loading conditions as that used in the experimental program.



Figure 6.31 – Mesh for Adebar Shell; SP7 (dimensions in millimetres)

The shell elements were subdivided into 25 equal thickness layers, resulting in a concrete layer thickness of approximately 12.5 mm. The longitudinal reinforcement was modelled discretely through the depth using four additional layers, and transverse reinforcement was smeared throughout the concrete layers.

All loads were applied in fixed proportions, using increments of approximately 2 % of that reported for the experimental failure loads. The results from the VecTor4 analyses are summarized below in Figure 6.32 and Table 6.9.



Figure 6.32 – Analytical versus Experimental Shear Capacities; Adebar Shells

The general trend resulting from the VecTor4 analyses presented in Figure 6.32 agrees well with the in-plane versus out-of-plane shear interaction behaviour observed in the experimental program. It is interesting to note that in the case of shell specimen SP4 which was subjected to additive compression force relative to the direction of out-of-plane shear, VecTor4 over-estimated the capacity of the shell. In specimens SP7 and SP9, which were subjected to additive tension, the resulting analytical failure loads were much more conservative. It is difficult to make any definitive conclusions regarding VecTor4 based on the relatively small data set considered from this study; however, the above noted tendencies agree with those observed from the VecTor4 analytical results pertaining to the Mattock beams which were subjected to combined out-of-plane shear and axial loading (see Section 6.2.4).

Overall, the VecTor4 strength calculations agree well with those reported from the experimental program. For the five specimens considered, the mean analytical-to-experimental capacity ratio was determined to be 0.97 with a coefficient of variation of approximately 14 % (see Table 6.9).

Specimen	<u>Experin</u>	<u>nental</u>	VecTo	<u>or4</u>	V., VT4 /		
	Vu,out-of-plane* (MPa)	V _{u,in-plane} (MPa)	Vu,out-of-plane* (MPa)	V _{u,in-plane} (MPa)	$v_{u, \sqrt{14}}$ $v_{u, \text{Exp}}$		
SP3	1.75	-	1.45	-	0.83		
SP4	2.15	8.60	2.45	9.80	1.14		
SP7	1.60	6.40	1.38	5.50	0.86		
SP8	-	16.80	-	18.00	1.07		
SP9	1.22	9.76	1.15	9.20	0.94		
				Mean = C.O.V. =	0.97 13.9 %		

Table 6.9 – Summary of Results for Adebar Shells

* calculated as V/bh

# Polak Shells

An experimental program performed by Polak and Vecchio (1994) involved the testing of four reinforced concrete shell elements. The elements were similar in geometry to those tested by Adebar; however, they were not as heavily reinforced. The program was focused on investigating the behaviour of elements subjected to various combinations of bending and membrane forces. With regard to assessing the performance of VecTor4, of particular interest were the loading conditions considered for shell elements SM3 and SM4 which were subjected to pure biaxial bending and combined in-plane shear and bending, respectively. A summary of the key material properties, the longitudinal reinforcement orientation, and the applied loading conditions considered for specimens is presented in Table 6.10.

Because the shells were loaded such that they experienced uniform stress conditions over the plane of the elements, a simple mesh consisting of only four elements was used to model the Polak shells (see Figure 6.33). Typically, a simple restraint condition as shown in the mesh illustration was considered, although the restraint conditions varied somewhat throughout the testing program. The elements were subdivided into twenty-five concrete layers to ensure that an accurate representation of the bending behaviour was achieved. Four additional layers were used to represent the longitudinal reinforcement, and out-of-plane reinforcement was treated as a property of the concrete layers.

The loads acting on the elements, which consisted of normal forces and/or bending moments, were applied uniformly along the free edges of the element.

Specimen	Applied Loading	<i>fc</i> ' (MPa)	$ ho_{x}^{*}$ (%)	f _{yx} (MPa)	ρ _y * (%)	f _{yy} (MPa)
SM1		47	1.25	425	0.42	430
SM2	$ \begin{array}{c} y  P \bigvee \stackrel{P}{\longrightarrow} \stackrel{M}{\longrightarrow} = 0.25 \text{ m} \\ & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & $	62	1.25	425	0.42	430
SM3	$M_{1} \bigoplus_{M_{2}} M_{2} \bigoplus_{M_{2}} M_{2} = 3.2$	56	1.25	425	0.42	430
SM4	$ \underset{P}{\overset{M}{\Rightarrow}} = 0.25 \text{ m} \overset{P}{\downarrow} \underset{M}{\overset{V}{\Rightarrow}} \underset{P}{\overset{V}{\Rightarrow}} \underset{M}{\overset{V}{\Rightarrow}} \underset{M}{\overset{V}{\to}} \underset{V}{\overset{V}{\to}} \underset{V}{\overset{V}{\to} \underset{V}{\overset{V}{\to}} \underset{V}{\overset{V}{\to}} \underset{V}{\overset{V}{\to} \underset{V}{\overset{V}{\to}} \underset{V}{\overset{V}{\to}} \underset{V}{\overset{V}{\to}} \underset{V}{\overset{V}{\to} } \underset{V}{\overset{V}{\to} \underset{V}{\overset{V}{\to}} \underset{V}{\overset{V}{\overset{V}{\to}} \underset{V}{\overset{V}{\overset{V}{\to}} \underset{V}{\overset{V}{\overset{V}{\to}} \underset{V}{\overset{V}{\overset{V}{\to}} \underset{V}{\overset{V}{\overset{V}{\to}} \underset{V}{\overset{V}{\overset{V}{\overset{V}{\to}} \underset{V}{\overset{V}{\overset{V}{\overset{V}{\overset{V}{\overset{V}{\overset{V}{\overset{V}{$	64	1.32	425	0.44	430

Table 6.10 - Summary of Polak Shell Specimens

* per layer of steel in each planar direction (same amount of steel on top & bottom) notes: specimens contained  $\rho_z = 0.08$  % out-of-plane reinforcement maximum aggregate size = 10 mm



Figure 6.33 – Mesh for Polak Shell; SM4 (dimensions in millimetres)

The analyses were performed using a load-step size of approximately 1 % of the experimental yield moment. The use of such a small load-step was selected to ensure that a reasonable level of resolution was achieved in the post-yield stages of the analyses.

A comparison between the observed and analytical moment versus curvature relationships for each of the shells is presented in Figure 6.34. Overall, the yielding moments, post-cracking stiffnesses, and post-yield stiffnesses from the VecTor4 analyses matched the experimental behaviour with high accuracy. In specimens SM3 and SM4, tension-stiffening contributions were somewhat overestimated; however, this discrepancy may be attributed to uncertainties regarding the assumed concrete tensile strengths. Additionally, the cracking moments of the specimens were also overestimated, also suggesting that the tensile strengths used in the VecTor4 analyses may have been higher than that of the actual test specimens.

In Figure 6.34d through f, the measured reinforcing bar strains from specimen SM4 have been compared with the results from the VecTor4 analysis. In general, the analytical results tended to underpredict the bar strains at any given load level; however, the overall trends matched the experimental behaviour quite well. Again, due to uncertainty regarding the tensile strengths of the high strength concretes used in the experimental program, the analytical results are considered to describe that measured from the experiments within a reasonable level of accuracy.

# Jaeger and Marti Slabs

In 2005 an international prediction competition focused on the analysis of large-scale reinforced concrete slabs subjected to out-of-plane shear was organized. Researchers were invited to submit predictions of the slab responses, and the results from the competition were later published in the American Concrete Institute's *Structural Journal* in 2009 (Jaeger and Marti, 2009b).

The four slab specimens forming the prediction competition (denoted as Slabs A, B, C, and D) were designed to investigate the impact of several influential variables. The main variables consisted of the overall slab thickness, *t*, which was either 200 or 500 mm, the orientation of the in-plane reinforcement,  $\varphi$ , which was aligned with the slab edges or was skewed by an angle 45 degrees (see Figure 6.35), and the addition of out-of-plane shear reinforcement,  $\rho_z$ .



Figure 6.34 – Analytical versus Experimental Responses; Polak Specimens

The main properties of the slabs considered in the competition are summarized in Table 6.11. The material properties of the concretes and reinforcing bars comprising the slabs are summarized in Table 6.12 and Table 6.13.

Slab	t (mm)	$\varphi$ - reinf. (degrees)	d _{eff} (mm)	${\displaystyle \rho_{n,long}}^1 \ (\%)$	${ ho_{t,long}}^1 \ (\%)$	ρ _{z,TEST-1} (%)	ρ _{z,TEST-2} (%)
А	200	45	156	1.812	1.812	0	0.611
В	200	0	162	1.745	0.873	0	0.309
С	500	45	390	1.812	1.812	0	0.311
D	500	0	405	1.745	0.873	0	0.308

Table 6.11 – Test Parameters; Jaeger and Marti Slabs

¹refer to Figure 6.36 for governing *n*-*t* coordinate system pertaining to each slab

Table 6.12 – Concrete Properties;

	Jaeg	er and M	arti	Table 6.15 – Keinforcing Bar Properties, Jaeger and Marti									
Slab	$f_c$ , 1 (MPa)	E _{cs} (MPa)	$f_t$ , ² (MPa)	$\begin{array}{cccc} f_t & f_t & Diameter / Slab & f_y & f_u & E_s \\ (MPa) & (MPa) & (MPa) & (MPa) & (MPa) \end{array}$		Es (MPa)	$\mathcal{E}_{sh}$ (x 10 ⁻³ )	$\mathcal{E}_u$ (x 10 ⁻³ )					
А	52.4	29,200	4.56	6 / B	554	563	209,400	1	22.8				
В	58.8	37,700	4.16	8 / A	517	579	207,000	<b></b> ¹	54.3				
С	52.4	31,800	4.29	12 / A	483	561	202,200	<b></b> ¹	67.4				
D	53.7	36,000	3.90	12 / B	540	623	200,000	14.9	95.5				
¹ evaluate	d at time of	f slab testin	g	14 / D	518	570	203,100	29.5	109				
² from dou	uble-punch	tests		20 / C	508	584	201,900	26.3	131				
note: maximum aggregate = 16 mm				30 / C, D	30 / C, D 539 619 209,8								
¹ coiled rainforcing steel													

coiled reinforcing steel

Each of the slabs was constructed with a region which contained no shear reinforcement, and each specimen was tested twice. Test 1 was used to evaluate the behaviour of the slab segments containing no out-of-plane reinforcement, and Test 2 was performed to investigate the behaviour of the slab segments with shear reinforcement.

In all cases, the in-plane reinforcing bars and the single-leg stirrups were fitted with welded steel anchor plates. For slabs A and C, which were constructed with skewed reinforcement, the inplane reinforcing bars were placed in four layers. For the slabs B and D, the reinforcement was placed in three layers (two longitudinal layers, and one layer in the width direction of the slab). The geometries of the slabs resulted in a uniform shear-span to slab thickness ratio of 3.2. The reinforcement layout for Slab C is presented in Figure 6.35.



Figure 6.35 – Reinforcement Layout for Slab C; (a) elevation, (b) plan (Jaeger and Marti, 2009b) (dimensions in millimetres)

A schematic of the loading conditions considered in the experimental program is presented in Figure 6.36. Note that all of the slabs were tested under one-way simply-supported conditions.



Figure 6.36 – Test Setup for Jaeger and Marti Slabs; (a) Slabs A and B, (b) Slabs C and D (Jaeger and Marti, 2009a) (dimensions in millimetres)

Because of the restraint conditions used in the experimental test setup, a finite element mesh consisting of the interior span and one exterior cantilever span was created in VecTor4. Taking advantage of symmetry, only half of the slab's width was considered. Thirty-six shell elements were used to model the slabs, with six shells comprising the longitudinal directions of both the interior and exterior spans. To accommodate the different loading conditions used in the testing program, the restraint conditions varied somewhat; however, most of the analyses were performed using a mesh similar to that presented in Figure 6.37. The shell elements were subdivided into thirty concrete layers. A clear cover of 20 mm was specified for the 200 mm thick slabs (Slabs A and B) and a clear cover of 50 mm was applied for the 500 mm thick slabs (Slabs C and D). The in-plane reinforcement was modeled using four additional steel layers for slabs with skewed reinforcement, and three additional layers for the slabs with non-skewed reinforcement. When present, out-of-plane reinforcement was treated as a property of the non-cover concrete layers.



Figure 6.37 – VecTor4 Mesh for Jaeger and Marti Slabs C and D

In the analyses, displacements were applied in one-millimetre increments for Slabs A and B, and in two-millimetre increments for Slabs C and D. The self-weight of the slabs was considered through the use of body-loads, and was applied prior to the displacement controlled loading. Default models and analysis options were considered for all analyses.

The computed responses from each of the eight tests are plotted alongside the experimental behaviours in Figure 6.38. It can be seen that for the first test performed on each slab, VecTor4 provided reasonable estimates of the behaviours observed in the laboratory; however, VecTor4 had the tendency to somewhat over-estimate the stiffnesses and strengths of the slabs with skewed reinforcement. In the case of the shear-reinforced slab segments (Test 2), the VecTor4



analyses produced reasonable estimates of the slab capacities and the initial stiffnesses, but greatly underestimated the ductility capacities of the shear strengthened slabs.

Figure 6.38 – Analytical versus Experimental Responses; Jaeger and Marti Slabs

All of the shear-reinforced slabs exhibited some form flexural-crushing in the laboratory. In the case of the slabs with skewed reinforcement, the resulting failure modes from the VecTor4 analyses agreed well with that observed in the laboratory; that is concrete crushing occurred on the compression face of the slab along the line of action, and lateral concrete crushing developed on the tension face of the slab resulting from compressive stresses developed by the skewed reinforcement configuration. The flexural concrete crushing failure modes were also accurately captured for Slabs B and D. However, VecTor4 was not capable of capturing the gradual

decaying of the load which was found to occur in the experiments. The underestimation of the slabs' ductilities may be, at least in part, due to the concentrated support restraints which were used in the VecTor4 model.

The moderators of the prediction competition ranked the eight contest submissions based upon their predictions of slab flexibility and moment capacity. The deviation of the analytical slab stiffness/flexibility from that observed in the laboratory was computed from (Jaeger and Marti, 2009b):

$$f = \frac{\tan^{-1}\left(\frac{K_{\exp}}{K}\right)}{\frac{\pi}{4}} - 1 \tag{6-27}$$

where,

K = predicted secant stiffness at peak moment, (N/mm);

 $K_{exp}$  = experimentally observed secant stiffness at peak moment, (N/mm).

Similarly, analytical deviation of the moment capacity of the slabs was calculated using Equation 6-28 (Jaeger and Marti, 2009b).

$$s = \frac{M}{M_{\rm exp}} - 1 \tag{6-28}$$

Ranking was done according to an *r*-value factor evaluated from the mean of the eight slab tests, and calculated for each submission:

$$r = \sqrt{f^2 + s^2} \tag{6-29}$$

The same evaluation procedures were used to assess the quality of the results obtained using VecTor4. From Figure 6.39 it can be seen that, in an averaged sense, VecTor4 achieved better estimates of the slab capacities and initial stiffnesses than any of the submissions entered in the competition. In fairness, however, it is worth reiterating that the ranking criteria used to judge the

quality of the results did not consider the post-beak behaviours, and as such, do not reflect the inaccurate ductility estimations which were computed using VecTor4.

Lastly, it is worth noting that the least accurate predictions submitted for consideration in the contest (group number 5) were those developed using a previous version of VecTor4. As noted in Section 6.1.2, older versions of VecTor4, APECS, and RASP demonstrated an inability to predict shear failures in members containing no shear reinforcement and had the tendency to overestimate the shear stiffness of elements subjected to combined flexure and shear. Based upon the results obtained from the current solution algorithm in VecTor4, it is evident that the capabilities of the program have improved significantly.



Figure 6.39 – Prediction Deviation from Experimental Behaviour; Jaeger and Marti Slabs

#### 6.2.6 R/FRC Specimens

The final set of tests used to corroborate VecTor4 under monotonic loading conditions involves the analysis of fibre-reinforced concrete elements containing conventional steel reinforcement (R/FRC). As one of the key objectives of this research was to implement a behavioural model which could effectively describe the behaviour of R/FRC elements, some level of verification is required.

An experimental program performed by Susetyo et al. (2011) involved the testing of eight fibrereinforced concrete panels which contained conventional steel reinforcement in only one of the orthogonal directions. The panels were tested under pure-shear monotonic loading conditions using the University of Toronto's Panel Element Tester. The purpose of the experimental program was to investigate the feasibility of using steel fibres to effectively replace minimum steel requirements prescribed in current code provisions.

The panels were 890 mm square by 70 mm thick and contained identical conventional reinforcement configurations with a reinforcement ratio of 3.31 % in the direction of the x-axis, and no conventional reinforcement in the y-direction. Several parameters were varied amongst the eight R/FRC panels: i) fibre volume,  $V_f$ , which ranged from 0.50 % to 1.50 %, ii) fibre type which included different fibre aspect ratios,  $l_f/d_f$ , fibre lengths,  $l_f$ , and fibre tensile strengths,  $f_{uf}$ , and iii) concrete compressive strength which nominally ranged from 50 MPa to 80 MPa. The experimental program also considered two panel elements which contained only conventional reinforcement; however, for the purpose of corroborating VecTor4's ability to analyze R/FRC elements, only the eight panels constructed from FRC have been considered. Table 6.14 presents a summary of the key properties pertaining to the R/FRC panel elements.

Panel	<i>fc</i> ' ^{1,2} (MPa)	$\rho_x^{\ 3}$ (%)	ρ _y (%)	V _f (%)	l _f (mm)	$d_f$ (mm)	$l_f/d_f$	f _{uf} (MPa)
C1F1V1	51.4	3.31	-	0.50	50	0.62	81	1050
C1F1V2	53.4	3.31	-	1.00	50	0.62	81	1050
C1F1V3	49.7	3.31	-	1.50	50	0.62	81	1050
C1F2V3	59.7	3.31	-	1.50	30	0.38	79	2300
C1F3V3	45.5	3.31	-	1.50	35	0.55	64	1100
C2F1V3	79.0	3.31	-	1.50	50	0.62	81	1050
C2F2V3	76.5	3.31	-	1.50	30	0.38	79	2300
C2F3V3	62.0	3.31	-	1.50	35	0.55	64	1100

Table 6.14 – Properties of Susetvo et al. Panels

¹ from compressive cylinder tests performed at panel test age ² maximum aggregate size = 10 mm ³ comprised of D8 bars with  $f_y = 552$  MPa &  $E_s = 225,000$  MPa

The panel elements were modelled using a mesh similar to that considered in the analyses of the Polak shell specimens (see Section 6.2.5). The use of four shell elements (2 x 2 grid) to model the panels ensured that the development of spurious zero-energy modes would not play a role in the analysis. Because the elements were subjected to in-plane stresses only, the elements were subdivided into two concrete layers. A third layer was used to represent the x-direction longitudinal reinforcement. Loads were applied as shear forces along the free edges of the elements resulting in a uniform in-plane shear stress condition. A load-step increment corresponding to an in-plane shear stress of 0.025 MPa was used for all panels. In Figure 6.40, the analytical shear stress-shear strain results have been compared against the experimental responses.



Figure 6.40 - Analytical versus Experimental Responses; Susetyo Panels

Based on the results presented in Figure 6.40 it can be seen that in the case of the Suestyo panels, the use of the SDEM to model the behaviour of the R/FRC panel elements resulted in a reasonably accurate representation of the shear stress-shear strain behaviours measured from the experimental program. Results from the experimental program and VecTor4 both indicate that failure of the panels occurs as a result of shear sliding, or an inability to maintain the developed shear stresses acting on the crack surface. In terms of shear strength predictions, the VecTor4 analyses resulted in a mean analytical-to-experimental strength ratio of 1.02 with a coefficient of variation of 12.5 %. When estimates of the elements' ductility are considered, it can be seen that a similar level of accuracy is obtained with a mean analytical-to-experimental ultimate shear-strain ratio of 1.02 with a coefficient of variation of 17.6 %. In general, when one considers the inherent difficulties in assessing the behaviour of reinforced concrete structures subjected to shear, the resulting VecTor4 calculations pertaining to the Susetyo panels do in fact validate the SDEM as a viable approach for modelling the behaviour of R/FRC elements.

# 6.3 Chapter Summary and Conclusions

In this chapter several of the key features and developments of the program were presented and discussed. Where applicable, practical examples were used to support provided discussions and to highlight the significance of some of the modifications and features added to the program.

The performance of VecTor4 was verified against test data from experimental programs involving structures subjected to monotonic loading conditions. The data set was focused primarily on shear-critical structures and elements. However, a wide range of structure types under varied loading conditions were considered including beams, slabs, plates, shells, and SFRC panel elements, many of which were subjected to three-dimensional loading conditions. The use of default material models and default analysis options were considered in all cases.

Overall, VecTor4 provided reliable and consistent strength estimates for the 80 structures/elements forming the monotonic verification data set with an overall analytical-to-experimental strength ratio of 1.02 and corresponding coefficient of variation of 13 %. This level of accuracy obtained for the capacity computations is believed to be acceptable; particularly when considering that the majority of the structures forming the verification data set were governed by shear-controlled failure mechanisms. Generally, less accurate estimates of the

structure displacement behaviours were computed; however, VecTor4 did provide reliable estimates of initial stiffnesses and the governing failure modes.

Throughout the development and verification of the monotonic analysis procedures, there were several issues identified that could benefit from some level of further development or study:

- i. The use of the parabolic out-of-plane shear strain profile resulted in significant improvement in the analysis of shear-critical members; however, in some instances VecTor4 still overestimated the capacity of shear-critical members subjected to combined flexure and out-of-plane shear. This was found to be particularly evident in members containing no out-of-plane shear reinforcement. It is recommended that the use of a more rational treatment of the out-of-plane shear behaviour be investigated, and potentially be added to the program as an alternative analysis option.
- ii. Many sectional analysis programs consider some form of out-of-plane shear strength modification to account for beneficial strength enhancements which are encountered in disturbed regions of structures. It was determined that the computed monotonic responses produced by VecTor4 were sufficiently accurate without the use of any disturbed region modification. However, in reference to the point noted above, the use of such strength enhanced regions may be required with the addition of a more sophisticated approach for modelling out-of-plane shear. Furthermore, structure ductility was typically underestimated in structures which contained sufficient levels of shear reinforcement and/or were ultimately controlled by flexural failure mechanisms. The consideration of the disturbed region strength enhancement may also aid in addressing this issue.
- iii. In several instances it was apparent that the post-peak response of structures experiencing concrete crushing was not captured well. In nearly all cases where crushing occurred, including those possessing low concrete strengths and those which were governed by flexural failure mechanisms, VecTor4 typically failed immediately after the onset of concrete crushing, even in structures which were experimentally shown to possess significant ductility.

# CHAPTER 7: DYNAMIC ANALYSIS

This chapter provides a summary of the dynamic analysis capabilities implemented within VecTor4. Justification of the selected dynamic analysis procedures and methodologies is provided, and the capabilities of the program are assessed through a dynamic verification study.

The verification study was comprised of linear elastic test cases which were compared with theoretical solutions, and the nonlinear analyses of RC structures tested experimentally. Two experimental studies were used to evaluate VecTor4's nonlinear dynamic analysis capabilities: i) shear-critical RC beams subjected to high-mass low-velocity impact loading conditions (Saatci and Vecchio, 2009a); and iv) the RC and R/FRC impact slabs forming the experimental study in this research program.

The chapter concludes with general findings regarding the performance of VecTor4 in the analysis of structures subjected to dynamic loading conditions. Recommendations for future development of the program are noted.

# 7.1 Theory and Implementation

This section presents an overview of the background and development of the dynamic analysis features implemented within VecTor4. Justification of the methods used to compute the dynamic system properties, to implement behavioural models which are unique to the performance of dynamic analyses, and to perform required numerical time integration is provided. Additionally, the methodology used to incorporate the dynamic analysis subroutines in a manner which is compatible with the VecTor4 solution algorithm is also presented.

# 7.1.1 Equation of Motion

Dynamic analysis capabilities within VecTor4 are accommodated through inclusion of additional force contributions which are not considered when performing static analyses: i) consideration of time-dependent loading conditions or forcing functions, ii) development of time-dependent inertial forces arising from mass accelerations, and iii) a solution algorithm which, unlike that used in static analyses, evaluates the behaviour of the structure through time, resulting in a response-time history as opposed to a single solution. Figure 7.1 illustrates the force

contributions considered in the static and dynamic analyses of a single degree-of-freedom (SDOF) system.



Figure 7.1 – SDOF Force Equilibrium (adapted from Saatci, 2007)

In the static system presented in Figure 7.1a, the applied force, p, is resisted entirely by the developed spring force of the system,  $f_S$ . In contrast, from Figure 7.1b, it can be seen that in a dynamic system, the time dependent force, p(t), is resisted by a combination of the system spring force,  $f_S(t)$ , the viscous damping force,  $f_D(t)$ , and inertial force  $f_I(t)$ . The resulting force equilibrium relationship represents the equation of motion of the system:

$$f_{S}(t) + f_{D}(t) + f_{I}(t) = p(t)$$
(7-1)

or

$$ku(t) + c\dot{u}(t) + m\ddot{u}(t) = p(t)$$
(7-2)

where,

u(t),  $\dot{u}(t)$ ,  $\ddot{u}(t)$  = displacement, velocity, and acceleration of the dof at time *t*, (m, m/s, m/s²); k = spring stiffness of the system, (N/m); c = damping coefficient of the system, (N-s/m); m = mass of the system, (N-s²/m).

For multi-degree-of-freedom (MDOF) systems, the equation of motion is expressed as:

$$[k]\{u(t)\} + [c]\{\dot{u}(t)\} + [m]\{\ddot{u}(t)\} = \{p(t)\}$$
(7-3)

where [k], [c], and [m] are square matrices representing the structural stiffness matrix, the structure damping matrix, and the structure mass matrix, respectively. The development and the solution of Equation 7-3 forms the basis of the dynamic analysis subroutine implemented in VecTor4. As such, a brief overview of the evaluation of the structural property matrices, namely [k], [c], and [m] is provided in the following section of the chapter.

## 7.1.2 Dynamic System Properties

This section of the thesis provides an overview of the methods used in VecTor4 to evaluate the required structural property matrices for dynamic analyses. Where appropriate, overviews of the general approaches used to characterize the system properties are presented and justification of selected methods or approaches is provided.

# 7.1.2.1 Mass Matrix, [*m*]

The method used to construct the mass matrix of a structure can significantly influence the computed analytical response. In reality, mass is distributed continuously throughout the structure and, as a result, inertial forces generated from structure accelerations also act in a distributed sense. In most software programs employing finite element analysis, one of two approaches for distributing the structure mass is usually considered: a consistent mass distribution method or a mass lumping method.

The consistent mass matrix is commonly used to represent the mass properties of a particular structure or element. In this method, the shape functions used in the derivation of the element stiffness matrix are also used to compute the element mass matrix; hence, the mass matrix is referred to as 'consistent.' Entries comprising the consistent finite element mass matrix are computed from:

$$m^e = \int_V N_i^T \rho N_j dV \tag{7-4}$$

where,

 $N_i$ ,  $N_j$  = nodal shape functions corresponding to dofs *i* and *j*, respectively;  $\rho$  = material density, (kg/m³); V = element volume, (m³).

DYNAMIC ANALYSIS

The most desirable attribute of the consistent mass matrix over other construction methods is the rationality of the development. Mass is distributed throughout the elements using the same assumptions which were used to derive element stiffness matrices and, as a result, mass matrices are element specific and directly accommodate irregular geometries or varied densities. However, there are several disadvantages associated with the use of consistent mass matrices, with the most significant disadvantages being related to their expensive computation requirements: the matrices possess non-zero off diagonal terms, are often sparsely populated, and typically require full bandwidths (Surana, 1978). In some instances, the use of a fully-populated consistent mass matrix will result in improved computational accuracy over simpler methods; however, in general, the marginal levels of increased accuracy rarely justify the required computational effort (Chopra, 2012).

To reduce computational demands arising from fully-populated consistent mass matrices, lumped mass matrices which treat mass as being concentrated at several discrete points coinciding with nodal locations are often considered. A lumped mass approach results in a diagonal mass matrix with all off diagonal terms being zero. Often rotational inertia is neglected, although some methods for incorporating rotary effects within diagonal matrices have been developed (Surana, 1978). In addition to the reduced storage and computational demands achieved from the use of a lumped mass matrix, the method is also well-suited for modal analysis techniques due to the decoupling of the equations of motion. However, in the case of complex high-order finite elements which often possess non-uniform or irregular geometries, appropriate mass lumping discretization schemes are not always apparent.

In VecTor4 a lumped mass approach proposed by Hinton et al. (1976), often referred to as *HRZ lumping* after the names of the authors, is considered. The *HRZ lumping* scheme provides a rational method for distributing the mass of high-order, regular or irregular, finite elements as concentrated masses applied at the node locations. The method consists of computing the diagonal terms of the consistent element mass matrix, and then scaling the terms such that the total mass of the element is preserved. The scaled terms are used to construct the diagonal of the mass matrix and all off diagonal terms are taken as zero. Minor modification of the relationship proposed by Rock and Hinton (1976) for the analysis of thick-plate elements results in the following:

$$m_{ii}^{e} = m_{i} = \frac{\left(\int N_{i}\rho N_{i}dV\right)\left(\int\rho dV\right)}{\int \left(\sum_{k=1}^{8} N_{k}\rho N_{k}\right)dv}$$
(7-5a)  
$$m_{ij}^{e} = 0$$
(7-5b)

where 
$$N_i$$
 is the nodal shape function corresponding to node *i*. The numerical integration required to evaluate Equation 7-5 is performed using a 3 x 3 Gauss quadrature integration rule and, for simplicity, rotational inertia contributions have not been considered. The resulting form of the elemental mass matrix for VecTor4's nine-noded 42 dof shell element takes the following¹:

1	$\begin{bmatrix} m_1 \end{bmatrix}$	0	0	0	0	0	0	0	0	0		0	0	0	0	0	0	0	
2	0	^{<i>m</i>} 1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	
3	0	0	<i>m</i> ₁	0	0	0	0	0	0	0	1   	0	0	0	0	0	0	0	
4	0	0	0	0	0	0	0	0	0	0	 	0	0	0	0	0	0	0	
5	_0	_0	0	0_	_0	0	_0	0	_0_	0	   +		_0	0	_0_	0	0_	_0_	
6	0	0	0	0	0	^m 2	0	0	0	0	   	0	0	0	0	0	0	0	
7	0	0	0	0	0	0	<i>m</i> ₂	0	0	0	 	0	0	0	0	0	0	0	
8	0	0	0	0	0	0	0	<i>m</i> 2	0	0		0	0	0	0	0	0	0	
9	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	
10	0	0	0	0	0	0	0	0	0	0	,   	0	0	0	0	0	0	0	
$\begin{bmatrix} m^e \end{bmatrix} = \cdot$						   					· ·	   						•	(7-6)
LJ·	.										·	1						•	(, )
•	•										•	1						•	
·	•					   					•							•	
•	<u> </u>					; +					+	; +						· 	
36	0	0	0	0	0	0	0	0	0	0	 	^m 8	0	0	0	0	0	0	
37	0	0	0	0	0	0	0	0	0	0		0	<i>m</i> 8	0	0	0	0	0	
38	0	0	0	0	0	0	0	0	0	0	1	0	0	<i>m</i> 8	0	0	0	0	
39	0	0	0	0	0	0	0	0	0	0	   	0	0	0	0	0	0	0	
40	0	_0	0	0_	_0	0	_0	0	_0_	0	; L		_0	0	_0_	0	0	_0_	
41	0	0	0	0	0	0	0	0	0	0			0	0	0	0	0	0	
42	0	0	0	0	0	0	0	0	0	0		0	0	0	0	0	0	0	

where the total mass of the element under consideration,  $M^e$ , is calculated as:

$$M^{e} = \sum_{i=1}^{8} m_{i}$$
(7-7)

¹ 5 dofs pertaining to each of the 8 perimeter nodes (3 translation, 2 rotation); 2 rotational dofs pertaining to the central 9th node (refer to Section 5.1.3)

Note that because the above mass matrix neglects rotational inertia effects, no mass contributions are assigned to the central ninth node of the shell element. This methodology agrees with that originally used by Rock and Hinton (1976) for the dynamic analysis of plate elements, and as shown in the following sections of this chapter, produces sufficiently accurate results.

# 7.1.2.2 Damping Matrix, [c]

Among the structural properties required for the assembly and evaluation of the equation of motion, the structural damping properties are perhaps the most difficult to quantify. In addition to the energy dissipated through the hystereses of the materials comprising the load-resisting elements of a structure, several additional mechanisms also dissipate energy, many of which cannot be explicitly considered in practical analysis methods: opening and closing of microcracks within concrete elements, frictional stresses developed between structural and nonstructural components or at element connections, and the development of appreciable stresses within nonstructural elements, to name a few (Chopra, 2012). Because of the wide range of sources which potentially contribute to the energy dissipation of a structure, damping characteristics are often specified using some form of global mathematical idealization.

Several procedures have been developed for constructing rational problem-dependent damping matrices that are both simple to attain and appropriate for the analysis of MDOF systems. The more common methods relate system damping to the calculated modal frequencies of the structure, and construct the damping matrix as a combination of one or more matrices which are related to the stiffness or mass of the system. Note that in this thesis the discussion regarding specific damping approaches is limited to classical damping procedures as they are used almost exclusively in the analysis of both linear and nonlinear systems (Charney, 2008).

Perhaps the most common approach for constructing the damping matrix of a structure is through the use of the procedure known as *Rayleigh's damping* method, after the work of Lord Rayleigh (1945). Also referred to as proportional damping, Rayleigh's method treats damping as a linear combination of stiffness proportional and mass proportional damping contributions using the following relationship:

$$[c] = a_o[m] + a_1[k] \tag{7-8}$$

where,

- $a_o$  = damping proportionality coefficient related to the structure mass, (s⁻¹);
- $a_1$  = damping proportionality coefficient related to the structure stiffness, (s).

The modal damping ratios are related to the frequencies through the following relation:

$$\xi_i = \frac{a_o}{2\omega_i} + \frac{a_1\omega_i}{2} \tag{7-9}$$

where  $\xi_i$  is the damping ratio (ratio of applied damping to critical damping) and  $\omega_i$  is the frequency of the *i*th mode of the system. Based upon the relationship presented in Equation 7-9, the proportionality coefficients  $a_o$  and  $a_1$  can be evaluated from the specification of two modal damping ratios, modes *i* and *j*:

$$\frac{1}{2} \begin{bmatrix} 1/\omega_i & \omega_i \\ 1/\omega_j & \omega_j \end{bmatrix} \begin{cases} a_o \\ a_1 \end{cases} = \begin{cases} \xi_i \\ \xi_j \end{cases}$$
(7-10)

which yields

$$a_1 = \frac{2\xi_j \omega_j - 2\xi_i \omega_i}{\omega_i^2 - \omega_i^2}$$
(7-11a)

$$a_{o} = 2\xi_{i}\omega_{i} - a_{1}\omega_{i}^{2} \tag{7-11b}$$

An example of the modal damping-frequency relationship developed from Rayleigh's method is presented in Figure 7.2a. It can be seen that the mass-proportional damping contributions most significantly impact the lowest frequency modes of the system and stiffness-proportional damping contributions tend to impact the higher modes. The combined effect of mass and stiffness damping contributions results in a damping matrix which is somewhat consistent with measured experimental data from actual structures (Chopra, 2012).

As an alternative to *Rayleigh damping*, a more generalized approach can be achieved using the procedure referred to as *Caughey damping* (Caughey, 1960). In comparison to *Rayleigh damping* where only two structural modes are explicitly controlled, *Caughey damping* provides analysts

with the ability to control any number of modal damping ratios (see Figure 7.2b). In the case where only two modal damping ratios are specified, the procedure breaks-down to the previously defined proportional damping matrix computed using Rayleigh's method (see Equation 7-8).



The alternative procedure is not as rigid as that used in Rayleigh's formulation; however, several numerical difficulties have been encountered when employing *Caughey damping* (Chopra, 2012): i) the relationships comprising the formulation are ill-conditioned, ii) if more than two modes of damping are specified, the method results in a fully populated damping matrix even though the structural stiffness matrix is typically banded, and iii) the method can produce negatively damped modes. Based on these factors, it was felt that Rayleigh damping was the most suitable approach for constructing the damping matrix in VecTor4.

In addition to the procedure used to develop the structural damping matrix, it has also been shown that the method of implementation can have a significant effect on the computed damping characteristics of nonlinear systems. In a numerical investigation performed by Charney (2008), the nonlinear analysis of a simple test structure was performed with three different implementation methods to develop the mass matrix using *Rayleigh damping*. With reference to Equation 7-8, implementation Method A considered a constant damping matrix with proportionality constants  $a_0$  and  $a_1$  computed from the eigenvalues of the elastic structure, and
considered the elastic stiffness matrix to construct the damping matrix. Method B also used elastic properties to compute the proportionality constants; however, it considered an updated stiffness matrix which accommodated material nonlinearity to compute the stiffness-dependent contribution of the damping matrix. Lastly, Method C used the updated stiffness matrix to constantly update the proportionality constants as well as the stiffness-dependent damping contributions. The three methods are summarized in Table 7.1.

Method	Stiffness used to compute coefficients	Stiffness used to compute damping	equation used to compute damping	
	$a_o, a_1$	matrix	matrix	
А	$K_{el}$	$K_{el}$	$[c]_{A} = a_{o,el}[m] + a_{1,el}[K_{el}]$	
В	$K_{el}$	$K_{nl}$	$[c]_{\mathrm{B}} = a_{o,el}[m] + a_{1,el}[K_{nl}]$	
С	$K_{nl}$	$K_{nl}$	$[c]_{C} = a_{o,nl}[m] + a_{1,nl}[K_{nl}]$	

Table 7.1 – Damping Matrix Computation Methods (adapted from Charney, 2008)

el = elastic; nl = nonlinear

From the results of the study, Charney found that the use of a constant damping matrix based entirely on the initial elastic stiffness of the structure (Method A) can have detrimental effects on the analysis, with damping ratios applied to the lowest frequency modes increasing several hundred percent over the course of the analysis. The use of Method B significantly reduced the amount of increased damping in the lower modes, however caution should be used in preventing excessive damping from occurring in the lower modes due to frequency shifts. Method C was most effective in maintaining a relatively constant level of damping; however, the approach may be impractical due to the repeated eigenvalue analysis which is required to update the constants  $a_0$ , and  $a_1$ . Lastly, Charney recommended that damping be provided primarily, if not entirely, through hysteretic mechanisms resulting from the structural response, reducing the impact of the inaccuracies and assumptions regarding the development of the damping matrix.

In VecTor4, damping of the structural response is primarily attributed to the hysteretic behaviour of the materials: the reinforcement hysteresis and the hysteretic response of the cracked concrete. The use of additional viscous damping provided from the inclusion of a damping matrix is required only as a means of providing numerical stability, and is not intended to serve as a significant source of energy dissipation. As such, the method of computing the damping matrix is likely of less significance in VecTor4 than in other programs which rely on dissipating energy

through the use of additional viscous damping alone. However, to minimize the development of increased artificial damping over the course of an analysis due to severe changes in the structural stiffness, the damping matrix in VecTor4 is computed using Charney's Method B (see Table 7.1). That is, the proportionality constants  $a_0$  and  $a_1$  are calculated once at the beginning of the analysis using the initial/elastic properties of the structure, but the damping matrix is actually computed on the basis of the continuously updated secant stiffness properties of the materials. Again, the use of more sophisticated methods which require significantly more computation (e.g., Method C in Table 7.1) are not justifiable for programs such as VecTor4 which consider artificial damping to enhance solution stability as opposed to modelling behaviour.

## 7.1.2.3 Stiffness Matrix, [k]

The stiffness matrix of the structure is computed in accordance with the DSFM, using the methodologies discussed previously in Chapter 5. By default, modification of the constitutive behaviours of the concrete and reinforcement materials to account for strain rate effects is considered (see Section 7.1.3); however, the general procedure for computing the stiffness matrix remains unchanged.

## 7.1.2.4 Load Vector, $\{p(t)\}$

The external load vector,  $\{p(t)\}$ , in the equation of motion consists of both time varying dynamic loads and constant static loads. Three general types of dynamic loading conditions can be considered in VecTor4: i) initial velocity and/or constant acceleration assignments to lumped masses, ii) time varying base accelerations applied globally to the structural masses, and iii) nodal force-time histories which can be specified for any number of nodes and can be used to represent any type of predefined impulsive loading scenario.

#### Initial Velocity Assignments

The use of initial mass velocity assignments is particularly useful in the analysis of mass impact loading scenarios. Under these types of loads the response of a structure can be highly sensitive to assumed or approximate user-defined impact force-time histories which are challenging to estimate, and are even difficult to accurately measure from experimental mass impact testing. As such, the use of initial mass velocity assignments provides a simple approach for modelling a dynamic mass impact loading scenario. Note that in this type of loading scenario, the assigned velocities are treated as the initial conditions of the dynamic problem as opposed to forces incorporated within the external load vector.

#### **Base** Accelerations

Ground motions resulting from seismic events can be considered in the analysis by way of userspecified ground accelerations. In this case, the time dependent load vector is created by replacing the ground motions with a series of effective seismic forces distributed throughout the structure, using the following relationship:

$$\{p(t)\} = \{p_{eff}(t)\} = -[m]\{r\}\ddot{u}_g(t)$$
(7-12)

where  $\ddot{u}_g(t)$  is the base acceleration at time *t*, and  $\{r\}$  is the dynamic coupling vector used to specify which dofs are influenced by the base accelerations (i.e., for ground motions in the global-*x* direction, only dofs with structural masses pertaining to translation in the global-*x* direction develop equivalent seismic forces). Ground motions in only one, two, or all three global directions acting simultaneously can be considered. The acceleration-time histories can be manually specified by the user or can be uploaded as digital earthquake records which are available from a number of online sources.

#### Constant Acceleration Assignments

Constant mass accelerations are treated in the same manner as that described above for base accelerations. They are used primarily to consider gravitational effects, but can be specified in any global direction.

#### Impulse Loads

User-defined impulse loads are particularly useful in cases where the external force-time history event is known. In such cases, time-dependent loads can be specified for any number of nodes, and can be used to represent highly nonlinear impulse responses such as those resulting from blast or impact. For example, consider the force-time history presented in Figure 7.3.

The series of user-defined discrete (F, t) points are used to develop the continuous external forcetime history response for a given node. Therefore, at any time, the nodal force can be computed and simply added to the nodal load vector,  $\{p(t)\}$ .



Figure 7.3 – User-Defined Force-Time History

#### 7.1.3 Strain Rate Effects

In general, the behaviours of structures subjected to high loading rates are not well understood, particularly in terms of the response at the material level. Many researchers have taken the stance that the mechanical properties of the materials comprising a structure are affected by the rate of loading. This viewpoint stems from experimental studies which have demonstrated that materials tested at high-strain rates exhibit increased load resistances when compared to results obtained from static tests. From these studies many relationships have been proposed to model the apparent strength increase of materials through what are commonly referred to as Dynamic Increase Factors (DIF). In the case of plain concrete, strain rate effects are typically considered through modification of the strength and stiffness parameters of the concrete, and in the case of steel reinforcing bars, increase factors are typically used to modify strength parameters only.

In a recent review of several models proposed for computing the DIF for plain concrete and steel reinforcement, Guner and Vecchio (2012) highlighted the large discrepancies amongst the various relationships which have been proposed over the last thirty years (see Figure 7.4).

Other researchers believe that the mechanical properties of the materials (e.g., concrete and steel) are independent of loading rate, and attribute apparent increases in strength to inertial effects leading to the development of confining stresses (Georgin and Reynouard, 2003; Cotsovos et al., 2008). However, Cotsovos et al. (2008) did suggest that the application of DIFs may serve as a practical modelling approach when coarse structure discretization and mass lumping schemes are employed.



Figure 7.4 – Comparison of Proposed Dynamic Increase Factors (Guner and Vecchio, 2012)

In VecTor4, the DIF approach is used to account for the apparent materials strength gains observed in experimental studies. Although VecTor4 does employ concrete confinement models, the relatively large element sizing and coarse mass lumping representation is not sufficient to accurately capture the beneficial inertial effects which some contend is responsible for the rate-dependent material characteristics. Recall that the layered shell elements used in VecTor4 allow mass to be distributed across the plane of the element; however, no mass distribution is provided through the thickness (i.e., out-of-plane direction) of the elements.

As noted above, numerous models have been proposed to calculate strain rate dependent DIFs for concrete and steel reinforcing bars. A number of these relationships have been implemented as analysis options within VecTor4. By default, the latest *fib* Model Code (*fib* MC 2010) provisions are used to compute the DIFs for concrete, and the model proposed by Malvar and Crawford (1998) is used for steel reinforcement. The initial selection of these models as default analysis options was primarily based on the successes achieved in the work of Guner and Vecchio (2012) which considered these strain-rate relationships, and employed a related modelling approach in the analysis of RC beams, columns, and frames under varied dynamic loading conditions.

In applying the rate dependent modifications, the strain rate,  $\dot{\varepsilon}$ , (see Equation 7-13) is computed in each of the three principal stress directions of the concrete, and in the direction of each of the steel reinforcing bar components.

$$\dot{\varepsilon} = \frac{\varepsilon_{i+1} - \varepsilon_i}{\Delta t} \tag{7-13}$$

where  $\varepsilon_i$  is the total strain at time  $t_i$ ,  $\varepsilon_{i+1}$  is the total strain at some time step later (time  $t_i + \Delta t$ ), where the time step length is  $\Delta t$ . The strain rates and resulting DIFs are computed and updated in the same iterative manner used in the general flow of the program.

## 7.1.3.1 Concrete DIF Model (fib MC 2010)

The provisions of the *fib* Model Code propose a series of strain rate dependent relationships for concrete loaded under tension or compression. The relationships are independent of concrete material properties.

#### Compression

The range of applicability for the *fib* provisions pertaining to concrete loaded in compression is specified as  $30 \cdot 10^{-6} \text{ s}^{-1} < |\dot{\epsilon}| < 300 \text{ s}^{-1}$ . For strain rates exceeding this applicable strain rate range, it has been assumed that the DIFs can be conservatively taken equal to those calculated for the maximum valid rate:  $|\dot{\epsilon}| = 300 \text{ s}^{-1}$ .

The dynamic increase factor pertaining to concrete compressive strength,  $DIF_{fc}$ , is computed as:

$$DIF_{fc} = \left(\frac{|\dot{\varepsilon}|}{30 \cdot 10^{-6}}\right)^{0.014} \qquad \text{for } |\dot{\varepsilon}| \le 30 \text{ s}^{-1} \tag{7-14a}$$

$$DIF_{fc} = 0.012 \left(\frac{|\dot{\varepsilon}|}{30 \cdot 10^{-6}}\right)^{1/3} \qquad \text{for } 30 \text{ s}^{-1} < |\dot{\varepsilon}| \le 300 \text{ s}^{-1}$$
(7-14b)

$$DIF_{fc} = 0.012 \left(\frac{300}{30 \cdot 10^{-6}}\right)^{1/3} \approx 2.6 \quad \text{for } |\dot{\varepsilon}| > 300 \text{ s}^{-1}$$
 (7-14c)

For the modulus of elasticity, the dynamic increase factor,  $DIF_{Ec}$ , is computed using:

$$DIF_{Ec} = \left(\frac{\left|\dot{\varepsilon}\right|}{30 \cdot 10^{-6}}\right)^{0.026} \qquad \text{for } 30 \cdot 10^{-6} \text{ s}^{-1} < \left|\dot{\varepsilon}\right| < 300 \text{ s}^{-1} \qquad (7-15a)$$

$$DIF_{Ec} = \left(\frac{300}{30 \cdot 10^{-6}}\right)^{0.026} \approx 1.5$$
 for  $|\dot{\varepsilon}| \ge 300 \text{ s}^{-1}$  (7-15b)

And for the strain corresponding to the peak compressive stress, the increase factor,  $DIF_{Ec}$ , is computed as:

$$DIF_{\varepsilon_0} = \left(\frac{\left|\dot{\varepsilon}\right|}{30 \cdot 10^{-6}}\right)^{0.02} \qquad \text{for } 30 \cdot 10^{-6} \text{ s}^{-1} < \left|\dot{\varepsilon}\right| < 300 \text{ s}^{-1} \qquad (7-16a)$$

$$DIF_{\varepsilon_0} = \left(\frac{300}{30 \cdot 10^{-6}}\right)^{0.02} \approx 1.4 \qquad \text{for } |\dot{\varepsilon}| \ge 300 \text{ s}^{-1}$$
 (7-16b)

## Tension

The range of applicability for concrete loaded in tension is specified as  $1 \cdot 10^{-6} \text{ s}^{-1} < |\dot{\varepsilon}| < 300 \text{ s}^{-1}$ . Again, as in the case of compressively loaded concrete, it is assumed that for strain rates in excess of 300 s⁻¹, the DIFs are computed using  $|\dot{\varepsilon}| = 300 \text{ s}^{-1}$ .

The dynamic increase factor for concrete tensile strength,  $DIF_{ft}$ , is computed as:

$$DIF_{ft} = \left(\frac{\left|\dot{\varepsilon}\right|}{1 \cdot 10^{-6}}\right)^{0.018} \qquad \text{for } \left|\dot{\varepsilon}\right| \le 10 \text{ s}^{-1} \qquad (7-17a)$$

$$DIF_{ft} = 0.0062 \left(\frac{|\dot{\varepsilon}|}{1 \cdot 10^{-6}}\right)^{1/3} \qquad \text{for } 10 \text{ s}^{-1} < |\dot{\varepsilon}| \le 300 \text{ s}^{-1} \tag{7-17b}$$

$$DIF_{ft} = 0.0062 \left(\frac{300}{1 \cdot 10^{-6}}\right)^{1/3} \approx 4.2 \qquad \text{for } |\dot{\varepsilon}| > 300 \text{ s}^{-1}$$
 (7-17c)

And the tensile modulus of elasticity dynamic increase factor,  $DIF_{Ect}$ , is computed using:

$$DIF_{Ect} = \left(\frac{\left|\dot{\varepsilon}\right|}{1 \cdot 10^{-6}}\right)^{0.026} \qquad \text{for } 1 \cdot 10^{-6} \text{ s}^{-1} < \left|\dot{\varepsilon}\right| < 300 \text{ s}^{-1} \qquad (7-18a)$$

$$DIF_{Ect} = \left(\frac{300}{1 \cdot 10^{-6}}\right)^{0.026} \approx 1.7$$
 for  $|\dot{\varepsilon}| \ge 300 \text{ s}^{-1}$  (7-18b)

The resulting DIF versus strain rate relationships from the *fib* MC 2010 model are summarized in Figure 7.5. From the figure, and Equations 7-14c and 7-17c, it is evident that the proposed formulation results in more significant strength enhancement for concrete loaded in tension than in compression (note the change of scale on the y-axis). Additionally, for both tensile and compressive loading, the computed rate enhancements for concrete subjected to very high loading rates (i.e.,  $|\dot{\varepsilon}| \ge 100 \text{ s}^{-1}$ ) are appreciably larger than those computed for concrete stiffness ( $DIF_{Ec}$ ,  $DIF_{co}$ ,  $DIF_{Ect}$ ). Lastly, it should be noted that the assumptions made regarding concrete strain rate effects for rates that exceed 300 s⁻¹ abruptly limit the increasing strength enhancement. As future formulations are developed, the implementation of alternative strain rate models with larger ranges of validity may be beneficial in the analysis of structures subject to highly impulsive loads.



Figure 7.5 – fib MC 2010 Concrete DIF

## 7.1.3.1 Steel Reinforcement DIF Model (Malvar and Crawford, 1998)

The default strain rate model implemented for the reinforcing bars is based on the relationships presented by Malvar and Crawford (1998). This model was developed based upon a comprehensive review of experimental data which was available at that time, and is based principally from formulations presented in Malvar (1998). The authors note that the model is applicable for  $1 \cdot 10^{-4}$  s⁻¹ <  $\dot{\epsilon}$  < 225 s⁻¹, and is valid for reinforcing bars with static yield stresses ranging from approximately 290 to 710 MPa. The resulting formulation is summarized below.

The dynamic increase factor for the yield stress of reinforcing bars, DIF_{fy}, is computed from:

$$DIF_{fy} = \left(\frac{\dot{\varepsilon}}{1 \cdot 10^{-4}}\right)^{0.074 - 0.040 \left(\frac{f_y}{414}\right)} \qquad \text{for } 1 \cdot 10^{-4} \text{ s}^{-1} < \dot{\varepsilon} < 225 \text{ s}^{-1}$$
(7-19a)

$$DIF_{fy} = \left(2250 \cdot 10^3\right)^{0.074 - 0.040 \left(\frac{f_y}{414}\right)} \qquad \text{for } \dot{\varepsilon} \ge 225 \text{ s}^{-1} \tag{7-19b}$$

And for ultimate bar stresses, the dynamic increase factor,  $DIF_{fu}$ , is calculated as:

$$DIF_{fu} = \left(\frac{\dot{\varepsilon}}{1 \cdot 10^{-4}}\right)^{0.019 - 0.009 \left(\frac{f_y}{414}\right)} \qquad \text{for } 1 \cdot 10^{-4} \text{ s}^{-1} < \dot{\varepsilon} < 225 \text{ s}^{-1}$$
(7-20a)

$$DIF_{fu} = \left(2250 \cdot 10^3\right)^{0.019 - 0.009 \left(\frac{f_y}{414}\right)} \qquad \text{for } \dot{\varepsilon} \ge 225 \text{ s}^{-1} \tag{7-20b}$$

where  $f_y$  is the static yield stress of the bar in units of MPa. Note that the dynamic increase factors applied to both the yield and the ultimate rebar stresses are functions of the yield stress as well as strain rate.

The Malvar and Crawford (1998) DIF versus strain rate relationship is presented in Figure 7.6. As was the case for the concrete model, the maximum DIF values are computed using the maximum strain rate within the specified applicable range (i.e.,  $\dot{\varepsilon} = 225 \text{ s}^{-1}$ ). Lastly, because the model formulations result in larger yield stress enhancements than ultimate stress enhancements, it is possible that the formulation will result in modified material properties with  $f_y > f_u$ .



Figure 7.6 - Malvar and Crawford (1998) Rebar DIF

# 7.1.4 Numerical Solution Method

Many solution procedures have been developed to evaluate the equation of motion (Equations 7-1 and 7-2) for MDOF systems. Most of these solution methods employ modal analysis techniques and, in general, are only applicable for the analysis of linear elastic systems. For the analysis of nonlinear systems, numerical time-stepping methods are much more robust and are essentially the only generally applicable approach (Clough and Penzien, 1993).

Time-stepping methods are direct integration procedures which can be used to evaluate the equation of motion at discrete intervals of time. Assumptions that, over a small increment of time (i.e., the length of a time-step), the physical properties of the structure are constant and the variation of acceleration is known form the basis of the methods. In VecTor4, three time-stepping procedures have been implemented: Newmark's constant average acceleration method (Newmark, 1959), Newmark's linear acceleration method (Newmark, 1959), and Wilson's Theta method (Wilson, 1973).

# Newmark's Constant Average Acceleration Method

The average acceleration method is an unconditionally stable procedure which possesses no inherent numerical damping. Any time interval can be considered in the analysis; however, the underlying assumption that the acceleration remains constant throughout the time step requires that small step sizes be used to obtain high levels of accuracy.

# Newmark's Linear Acceleration Method

The linear acceleration method is a conditionally stable procedure which possesses no inherent numerical damping. A time step size which is less than  $0.551 \cdot T_N$ , where  $T_N$  is the natural period of the highest frequency mode of the system, is required to maintain stability. The linear acceleration has been shown to outperform the average acceleration method in terms of solution accuracy; however, the strict stability requirement limits its application for the analysis of large computation intensive MDOF systems.

# Wilson's Theta Method

Lastly, Wilson's Theta method is, to some degree, an unconditionally stable version of Newmark's linear acceleration method. The method is developed on the basis that the acceleration varies linearly over an extended time interval. In contrast to Newmark's average and linear acceleration methods, Wilson's method produces numerical algorithmic damping which is dependent on the time step size. For some applications this form of numerical damping can be considered advantageous as it has the tendency of damping out spurious high frequency modes. However, a major drawback of the method is that it has been reported to severely overestimate the responses of high-velocity problems during the initial stages of the analysis (Paultre, 2011).

Based upon the noted characteristics of the three numerical solution procedures implemented in VecTor4, Newmark's constant average acceleration method is the most suitable method to be used as a general default dynamic analysis option. As such, further discussion regarding implementation and the numerical solution algorithm will be limited to Newmark's method. However, it is worth noting that all three methods employ similar solution algorithms.

# 7.1.4.1 Incremental Equation of Motion

In conforming to the time increment based numerical solution procedures, it is convenient to express the governing equation of motion in the form of an incremental equation of motion. Recalling the equation of motion previously presented in Equation 7-1, at time t the equilibrium of forces for a dynamic system can be expressed as:

$$f_{S}(t) + f_{D}(t) + f_{I}(t) = p(t)$$
(7-1)

After some small change in time, denoted by  $\Delta t$ , the force equilibrium of the system becomes:

$$f_{S}(t + \Delta t) + f_{D}(t + \Delta t) + f_{I}(t + \Delta t) = p(t + \Delta t)$$
(7-21)

The incremental equation of motion is then developed by subtracting Equation 7-1 from 7-21:

$$\Delta f_{S}(t) + \Delta f_{D}(t) + \Delta f_{I}(t) = \Delta p(t)$$
(7-22)

where

$$\Delta f_s(t) = f_s(t + \Delta t) - f_s(t) = k(t)\Delta u \tag{7-23a}$$

$$\Delta f_D(t) = f_D(t + \Delta t) - f_D(t) = c(t)\Delta \dot{u}$$
(7-23b)

$$\Delta f_I(t) = f_I(t + \Delta t) - f_I(t) = m\Delta \ddot{u}$$
(7-23c)

$$\Delta p(t) = p(t + \Delta t) - p(t) \tag{7-23d}$$

From the incremental force expressions presented in Equation 7-23, the incremental equation of motion presented in Equation 7-22 can be expressed as:

$$k(t)\Delta u + c(t)\Delta \dot{u} + m\Delta \ddot{u} = \Delta p(t) \tag{7-24}$$

For reference, the time at the beginning of the time interval is referred to as  $t_i$  and the time at the end of the interval is referred to as  $t_{i+1}$ . This yields the following definitions for the incremental displacement,  $\Delta u$ , the incremental velocity,  $\Delta \dot{u}$ , and the incremental acceleration,  $\Delta \ddot{u}$ :

$$\Delta t = t_{i+1} - t_i \tag{7-25a}$$

$$\Delta u = u_{i+1} - u_i \tag{7-25b}$$

$$\Delta \dot{u} = \dot{u}_{i+1} - \dot{u}_i \tag{7-25c}$$

$$\Delta \ddot{u} = \ddot{u}_{i+1} - \ddot{u}_i \tag{7-25d}$$

#### Newmark's Method

Newmark's solution method, which encompasses the average and the linear acceleration methods, is the most common solution approach employed in the dynamic analysis of linear and nonlinear systems. The general form of Newmark's method is based on the following equations:

$$\dot{u}_{i+1} = \dot{u}_i + \left[ (1 - \gamma) \Delta t \right] \ddot{u}_i + (\gamma \Delta t) \ddot{u}_{i+1}$$
(7-26)

$$u_{i+1} = u_i + (\Delta t)\dot{u}_i + \left[\left(\frac{1}{2} - \beta\right)\Delta t^2\right]\ddot{u}_i + \left[\beta\Delta t^2\right]\ddot{u}_{i+1}$$
(7-27)

Parameters  $\beta$  and  $\gamma$  define the variation of the acceleration over the time step and influence the stability and accuracy of the numerical solution. Typically, the value of  $\gamma$  is taken as 1/2, and

values of  $\beta$  in the range of 1/6 to 1/4 yield solutions which are both accurate and stable (Chopra, 2012). The following stability limit can be used to determine the critical time step increment for all values of  $\beta$  and  $\gamma$ :

$$\Delta t \le \frac{\Delta t_{cr}}{T_N} = \frac{1}{2\pi \sqrt{\gamma/2 - \beta}}$$
(7-28)

The constant average acceleration method considers integration parameter values of  $\gamma = 1/2$  and  $\beta = 1/4$ . It can be seen that when these parameters are used to compute the critical time step for any system it results in  $\Delta t_{cr}/T_N = \infty$ ; this implies that the method is stable for all value of  $\Delta t$  as noted in the previous section.

In its general form, Newmark's method is referred to as being an implicit solution method, meaning that the right-hand side of Equations 7-26 and 7-27 are functions of unknown motion parameters at the end of the time step. As such, an iterative computation scheme is required for the time integration technique; however, in VecTor4 this is of little consequence since an iterative procedure is employed to accommodate material nonlinearity. The following presentation of the governing equations used in the time integration scheme is based on the development provided by Saatci (2007).

Substitution of Equation 7-25 into Equations 7-26 and 7-27 yields the following set of equations which are functions of the incremental motion parameters ( $\Delta u$ ,  $\Delta \dot{u}$ ,  $\Delta \ddot{u}$ ) and the values of the known motion parameters ( $u_i$ ,  $\dot{u}_i$ ,  $\ddot{u}_i$ ) at the beginning of the time step:

$$\Delta \dot{u} = (\Delta t)\ddot{u}_i + (\gamma \Delta t)\Delta \ddot{u} \tag{7-29}$$

$$\Delta u = \left(\Delta t\right)\dot{u}_{i} + \left(\frac{\Delta t}{2}\right)\ddot{u}_{i} + \left(\beta\Delta t^{2}\right)\Delta\ddot{u}$$
(7-30)

Equation 7-30 can be rearranged to directly solve for the incremental acceleration as a function of the incremental displacement, and values of the acceleration and velocity from the start of the time step in Equation 7-31.

$$\Delta \ddot{u} = \left(\frac{1}{\beta \Delta t^2}\right) \Delta u - \left(\frac{1}{\beta \Delta t}\right) \dot{u}_i - \left(\frac{1}{2\beta}\right) \ddot{u}_i$$
(7-31)

Substitution of Equation 7-31 into 7-29 results in an equation which is a function of the same motion parameters as Equation 7-31 (i.e., a function  $\Delta u$ ,  $\dot{u}_i$ ,  $\ddot{u}_i$ ):

$$\Delta \dot{u} = \left(\frac{\gamma}{\beta \Delta t}\right) \Delta u - \left(\frac{\gamma}{\beta}\right) \dot{u}_i - \left(\frac{\gamma \Delta t}{2\beta}\right) \ddot{u}_i$$
(7-32)

It can be seen that Equations 7-31 and 7-32 are now functions of known motion parameters  $\dot{u}_i$ ,  $\ddot{u}_i$  from the beginning of the time interval, and are also functions of the unknown incremental displacement,  $\Delta u$ . This form is convenient for structural analysis problems as displacements are commonly treated as unknown variable.

Substitution of the incremental motion parameters (Equations 7-31 and 7-32) into the governing incremental equation of motion presented in Equation 7-24 results in the following relationship:

$$\left(\frac{1}{\beta\Delta t^{2}}m + \frac{\gamma}{\beta\Delta t}c_{i} + k_{i}\right)\Delta u = \Delta p_{i} + \left(\frac{1}{\beta\Delta t}m + \frac{\gamma}{\beta}c_{i}\right)\dot{u}_{i} + \left[\frac{1}{2\beta}m + \Delta t\left(\frac{\gamma}{2\beta} - 1\right)c_{i}\right]\ddot{u}_{i}$$
(7-33)

Equation 7-33 can be further reduced to the more familiar form:

$$\hat{k}_i \Delta u = \Delta \hat{p}_i \tag{7-34}$$

where

$$\hat{k}_{i} = \left(\frac{1}{\beta\Delta t^{2}}m + \frac{\gamma}{\beta\Delta t}c_{i} + k_{i}\right)$$
(7-35)

$$\Delta \hat{p}_{i} = \Delta p_{i} + \left(\frac{1}{\beta \Delta t}m + \frac{\gamma}{\beta}c_{i}\right)\dot{u}_{i} + \left[\frac{1}{2\beta}m + \Delta t\left(\frac{\gamma}{2\beta} - 1\right)c_{i}\right]\ddot{u}_{i}$$
(7-36)

Because  $\hat{k}_i$  and  $\Delta \hat{p}_i$  are comprised of values which are known at the beginning of each time step, the only system unknown is the incremental displacement, which can be computed as:

$$\Delta u = \frac{\Delta \hat{p}_i}{\hat{k}_i} \tag{7-37}$$

Once the value of the incremental displacement is determined, the motion parameters pertaining to the incremental velocity and the incremental acceleration can be updated using Equation 7-31 and 7-32.

## 7.1.4.2 Implementation of the Direct Integration Method

As discussed previously in Chapter 5, VecTor4 uses a total load, secant stiffness solution method. The structural stiffness matrix is developed on the basis of secant stiffnesses which change throughout the analysis as a result of material nonlinearity. The load vectors are assembled using the total loads acting on the structure, and the computed nodal displacements represent total displacements of the structure.

To accommodate the total load secant stiffness solution technique, the incremental equation of motion presented in the previous subsection has been recast by Saatci and Vecchio (2009b) such that total loads and secant stiffness properties may be considered:

$$\overline{k}_{i+1}u_{i+1} + c_{i+1}\dot{u}_{i+1} + m\ddot{u}_{i+1} = p_{stat} + p_{i+1}$$
(7-38)

where  $p_{stat}$  is the constant static load, and  $\overline{k}_{i+1}$  represents the secant stiffness at time  $t_{i+1}$ . It should be noted however, that VecTor4 updates the damping based on the current stiffness conditions of the structure (see Section 7.1.2.2), which differs from the constant damping matrix considered in the formulation presented by Saatci and Vecchio.

When Newmark's first equation (Eq. 7-26) is substituted into Equation 7-38, it takes the following form:

$$\overline{k}_{i+1}u_{i+1} + c_{i+1}\left(\dot{u}_i + \Delta t\ddot{u}_i + \gamma\Delta t\Delta \ddot{u}\right) + m\left(\ddot{u}_i + \Delta \ddot{u}\right) = p_{stat} + p_{i+1}$$
(7-39)

Substitution of Equation 7-31 into 7-39, and rearrangement into a matrix format, yields the following:

$$\begin{bmatrix} \begin{bmatrix} \overline{k}_{i+1} \end{bmatrix} + \frac{\begin{bmatrix} m \end{bmatrix} + \begin{bmatrix} c_{i+1} \end{bmatrix} \lambda \Delta t}{\beta \Delta t^2} \end{bmatrix} \{u_{i+1} \} =$$

$$\{p_{stat}\} + \{p_{i+1}\} + \frac{\begin{bmatrix} m \end{bmatrix} + \begin{bmatrix} c_{i+1} \end{bmatrix} \lambda \Delta t}{\beta} \begin{bmatrix} \frac{\{u_i\}}{\Delta t^2} + \frac{\{\dot{u}_i\}}{\Delta t} + \frac{\{\ddot{u}_i\}}{2} \end{bmatrix} - \begin{bmatrix} c_{i+1} \end{bmatrix} \begin{bmatrix} \{\dot{u}_i\} + \Delta t \{\ddot{u}_i\} \end{bmatrix} - \begin{bmatrix} m \end{bmatrix} \{\ddot{u}_i\}$$

$$(7-40)$$

Equation 7-40 can be further reduced to the familiar relation:

$$\left[\left[\overline{k}_{i+1}\right] + \left[k^*\right]\right] \left\{u_{i+1}\right\} = \left\{p_{stat}\right\} + \left\{p^*\right\}$$
(7-41)

where

$$\left[k^*\right] = \frac{\left[m\right] + \left[c_{i+1}\right] \gamma \Delta t}{\beta \Delta t^2} \tag{7-42}$$

$$\left\{ p^* \right\} = \left\{ p_{i+1} \right\} + \frac{\left[ m \right] + \left[ c_{i+1} \right] \gamma \Delta t}{\beta} \left[ \frac{\left\{ u_i \right\}}{\Delta t^2} + \frac{\left\{ \dot{u}_i \right\}}{\Delta t} + \frac{\left\{ \ddot{u}_i \right\}}{2} \right] - \left[ c_{i+1} \right] \left[ \Delta t \left\{ \dot{u}_i \right\} + \left\{ \ddot{u}_i \right\} \right] - \left[ m \right] \left\{ \ddot{u}_i \right\}$$
(7-43)

Equations 7-41 through 7-43 allow Newmark's direct integration method to be used in a total load secant stiffness solution method, and in a manner that conforms to the existing solution algorithm within VecTor4.

#### 7.2 Dynamic Loading: Verification

This section of the thesis presents a series of verification studies carried out to assess the performance of VecTor4 under dynamic loading conditions. Data from experimental test programs as well as closed-form theoretical solutions were included in the verification study. As was the case in the monotonic verification study, the emphasis was focused on performing analyses which required the use of VecTor4's thick-shell formulation.

## 7.2.1 Linear Elastic Verification

In the following subsection the results from a series of linear elastic analyses performed using VecTor4 are presented. The analyses were used to corroborate the implemented dynamic subroutines, and to verify the computed responses from simple test scenarios on beam and plate elements. Three analyses considering a single degree-of-freedom (SDOF) cantilever beam were

performed to evaluate the structure response under different loading scenarios: i) the free vibration response resulting from an initial mass velocity assignment, ii) the forced vibration response resulting from the application of an arbitrary impulse event, and iii) the displacement-time history response resulting from ground motions recorded during a seismic event in California¹. Additionally, eigenvalue analyses of MDOF simply-supported plate elements were performed to verify the applicability of the HRZ mass lumping scheme implemented in VecTor4 (refer to Section 7.1.2.1).

#### 7.2.1.1 SDOF Testing

The cantilever beam forming the SDOF test structure was modelled using four shell elements in VecTor4. Each of the shell elements were subdivided into fifty uniform thickness layers to ensure that an accurate stiffness representation of the structure was calculated. A mass of 500 kg was applied to the tip of the cantilever and was permitted to translate in the global-z direction only. No additional viscous damping was considered in any of the linear elastic analyses. The VecTor4 mesh, the structure geometry, and the structure properties are summarized in Figure 7.7a.

### Free Vibration Response

The response of the linear elastic test structure was evaluated under a prescribed initial mass velocity of 1.0 m/s. For comparative purposes, the closed-form theoretical free vibration response of the beam was computed using:

$$u(t) = \frac{\dot{u}_o}{\omega_n} \sin \omega_n t \tag{7-44}$$

where,

u(t) = tip displacement of the beam at time t, (m);

 $\dot{u}_{o}$  = initial mass velocity, (m/s);

 $\omega_n$  = natural frequency of the cantilever structure, (rads/s).

¹ ground motions recorded in Santa Monica, C.A., U.S.A. during the 1994 Northridge earthquake digital record obtained from: <u>http://peer.berkeley.edu/smcat/</u> (CDMG Station 25438/STM 090)



The VecTor4 analysis was performed using Newmark's average acceleration method with a time step of 0.0005 seconds. The computed results from the FE analysis were output at 0.005 second increments. The free vibration responses computed from the closed-form equation and the VecTor4 analyses are plotted in Figure 7.7b. From the figure it can be seen that good agreement between the two solutions was achieved, with maximum discrepancies in the displacement amplitude being in the order of 1.0 %.

### Forced Vibration Response

The response of the test beam under impulsive loading conditions was also investigated. The theoretical solution was computed using closed-form recurrence formulas developed directly from the equation of motion for LE SDOF systems (Chopra, 2012). For excitation functions which can be represented exactly through linear interpolation (i.e., forcing functions comprised of linear segments), the exact theoretical solution for linear elastic systems can be computed using this method.

The test case impulse consisted of an initial ramp up to a maximum applied force of 140 kN, followed by an irregular force descent which extended into the negative loading regime. After 150 milliseconds the applied loading was held constant at a value of -40 kN. The specified nodal impulse is presented in Figure 7.8a. Note that the impulse force-time history varies linearly between the noted (F, t) data points.



Figure 7.8 – Linear Elastic SDOF Beam; Forced Vibrations

The theoretical response developed from the recurrence formulas is compared with the response computed using VecTor4 in Figure 7.8b. As noted above, the VecTor4 analysis was performed using Newmark's average acceleration method with a time step of 0.0005 seconds, and the results were output at increments of 0.005 seconds. It can be seen that the response obtained from VecTor4 closely matches that computed from theory.

#### **Base** Accelerations

The last test-case investigates the cantilever's response under ground motions recorded in Santa Monica during the 1994 Northridge earthquake. The base acceleration record used as input in the analyses is presented in Figure 7.9a. The displacement-time history behaviour of the cantilever was computed using VecTor4 and, as in the case of the impulse verification, was also evaluated using closed-form recurrence formulas. From Figure 7.9b and Figure 7.9c, it can be seen that the results from VecTor4 agree well with the closed-form theoretical response.

The computed VecTor4 responses closely match the closed-form theoretical solutions for all of the linear elastic SDOF verification tests, suggesting that the implemented dynamic analysis subroutines pertaining to the construction of the dynamic load vectors and the numerical time integration procedures function properly.



Figure 7.9 – Linear Elastic SDOF Beam; Base accelerations

## 7.2.1.2 MDOF Testing

In addition to the SDOF analyses, a brief investigation regarding the computed behaviour of multi degree-of-freedom (MDOF) linear elastic plates has been performed. In contrast to the SDOF analyses summarized in the previous section, the MDOF analyses are required to assess the mass lumping procedure implemented within VecTor4.

The eigenvalue analysis of two square simply-supported linear elastic plate elements was performed using VecTor4 and the results have been compared with that of closed-form theoretical solutions. The selected test plates were identical in all aspects with the exception that one plate was dimensioned as a thick-plate with a span-to-thickness ratio (a/h) of 5, and one plate was dimensioned as a thin-plate with a/h equal to 100. The common plate properties were taken as the following: modulus of elasticity, E = 200,000 MPa; span length, a = 1 metre; Poisson's ratio, v = 0.30; and the density,  $\rho = 2,400$  kg/m³.

The results from the VecTor4 analyses have been compared with solutions calculated using both thin- and thick-plate theories. In accordance with thin-plate theory, a Navier type solution (Navier, 1823) which is valid for square simply-supported plates was used to compute the theoretical eigenvalues,  $\omega$ , for each integer pair m, n:

$$\omega_{m,n} = \frac{\pi^2}{a^2} \sqrt{\frac{D}{\rho h}} \left( m^2 + n^2 \right) \tag{7-45a}$$

where

$$D = \frac{Eh^3}{12(1-\nu^2)}$$
(7-45b)

Additionally, a closed-form thick-plate solution presented by Hashemi and Arsanjani (2005) developed on the basis of Mindlin plate theory was also considered. A simplified version of the closed-form Mindlin plate solution which is valid for square simply-supported plates is presented in Equation 7-46:

$$\omega_{m,n} = \frac{\beta}{a^2} \sqrt{\frac{D}{\rho h}}$$
(7-46a)

where

$$\beta_{m,n}^{2} = \frac{72\nu_{1}K^{2}}{(h/a)^{4}} \left\{ 1 + \frac{(h/a)^{2} \tilde{\beta}}{12} \left( 1 + \frac{1}{\nu_{1}K^{2}} \right) - \sqrt{\left[ 1 + \frac{(h/a)^{2} \tilde{\beta}}{12} \left( 1 + \frac{1}{\nu_{1}K^{2}} \right) \right]^{2} - \frac{(h/a)^{4} \tilde{\beta}^{2}}{36\nu_{1}K^{2}} \right\}}$$
(7-46b)

and

$$\tilde{\beta} = \pi^2 \left( m^2 + n^2 \right)$$
(7-46c)
(7-46d)
(7-46d)

m, n = integers (e.g., m = 1, 2, 3...) representing the number of half-waves of the mode shape;

D = flexural rigidity of the plate, (N-m);

 $K^2$  = shear correction factor (*taken as* 1.0 *in this case*).

Note that the shear correction factor used to accommodate non-uniform out-of-plane shear strain distributions has been neglected (i.e., taken as 1.0), and for the purpose of providing a direct comparison, the VecTor4 analyses were performed using an assumed uniform/constant out-of-plane shear strain distribution.

For both test plates, three mesh densities were considered in the VecTor4 analyses: a  $3 \times 3$  grid of elements, a  $6 \times 6$  grid, and a  $12 \times 12$  grid which were used to model the full dimensions of the plates. In all cases the elements were subdivided into fifty equal thickness concrete layers to ensure that accurate representations of the plates' sectional stiffnesses were achieved. The results from the eigenvalue analyses for the thin-plate and the thick-plate are presented in Table 7.2 and Table 7.3, respectively.

From Table 7.2, it can be seen that the computed VecTor4 modal frequencies closely match the theoretical results computed for the first nine modes of the thin-plate. Even with the coarsest mesh considered (3 x 3 grid of elements representing the full plate) seven of the first nine modal frequencies were computed within approximately 5 % of the theoretical solution. The VecTor4 analyses performed with increased mesh densities resulted in nearly perfect agreement with theory. Additionally, it can be seen that the theoretical solutions calculated using Navier and Mindlin theories are essentially the same in the case of the thin-plate.

			Navier	Mindlin	VecTor4	VecTor4	VecTor4
Mode	m	n	thin pl.	thick pl.	3 x 3 (*)	6 x 6 (*)	12 x 12 (*)
1	1	1	545	545	530 (0.972)	541 (0.992)	543 (0.996)
2, 3	2	1	1363	1362	1350 (0.990)	1358 (0.996)	1359 (0.997)
4	2	2	2181	2178	2108 (0.966)	2174 (0.997)	2170 (0.995)
5,6	3	1	2726	2722	3189 (1.170)	2719 (0.997)	2719 (0.997)
7, 8	3	2	3544	3537	3447 (0.972)	3524 (0.994)	3524 (0.994)
9	3	3	4908	4894	4658 (0.949)	4841 (0.986)	4871 (0.992)
				Mean * :	1.00	0.99	1.00
				C.V. (%) :	8.24	0.41	0.17

Table 7.2 – Computed Modal Frequencies  $\omega_{m,n}$  for Thin-Plate (a/h) = 100

* ratio of VecTor4 solution to theoretical Navier thin-plate solution: VT4/Navier

Examination of the results from the thick-plate analyses (see Table 7.3) show that VecTor4 did not compute the theoretical solution with the same high level of precision that was achieved in the case of the thin-plate analyses. Even with the densest mesh considered, the VecTor4 computed fundamental frequency of the thick-plate differed from theory by nearly 4 %, and the frequencies of modes 5 and 6 differed by more than 9 %. Additionally, as was the case in the analysis of the thin-plate, limited improvement in solution accuracy was obtained as a result of increasing the mesh density.

			Navier	Mindlin	VecTor4	VecTor4	VecTor4
Mode	m	n	thin-pl.	thick-pl.	3 x 3 (*)	6 x 6 (*)	12 x 12 (*)
1	1	1	10906	9779	9364 (0.958)	9413 (0.963)	9419 (0.963)
2, 3	2	1	27265	21651	21270 (0.982)	21726 (1.003)	21801 (1.007)
4	2	2	43623	31551	29048 (0.921)	31353 (0.994)	31637 (1.003)
5,6	3	1	54529	37421	33087 (0.884)	33781 (0.903)	33963 (0.908)
7, 8	3	2	70888	45428	40380 (0.889)	45333 (0.998)	46166 (1.016)
9	3	3	98152	57207	55407 (0.969)	56100 (0.981)	58016 (1.014)
				Mean * :	0.93	0.97	0.99
				C.V. (%) :	4.49	3.87	4.33

Table 7.3 – Computed Modal Frequencies  $\omega_{m,n}$  for Thick-Plate (a/h) = 5

* ratio of VecTor4 solution to theoretical Mindlin thick-plate solution: VT4/Mindlin

From the linear elastic plate analyses performed, it is evident that the mass lumping scheme implemented in VecTor4 serves as a reasonable approach for distributing the mass of the complex high-order shell elements employed in VecTor4. Even with use of a coarse finite element mesh, high levels of agreement were obtained between the theoretical and the computed results for the thin-plate test scenario. In the case of the thick-plate test, VecTor4 was unable to

achieve the same high level of precision for the computed modal frequencies. The errors in the VecTor4 thick-plate results are likely attributed, at least in part, to the rotational inertia contribution in the lumped mass matrix. Previous studies have shown that rotary inertial typically has little effect on the behaviour of thin-plates but can significantly influence the analytical results of thick-plates (Surana, 1978).

## 7.2.2 Analysis of Slab Test Specimens

One of the primary objectives of this research program was to perform a series of controlled experimental tests involving RC and R/FRC slabs subjected to high-mass low-velocity impacts. In addition to characterizing the performance of the test specimens under the highly dynamic loading condition, the testing program was also focused on obtaining a high-quality digital data set which would be suitable for verifying currently available and future developed numerical tools and analytical procedures. This section of this thesis presents the analysis of the slab tests undertaken with VecTor4. Discussions regarding the modelling approach, the selected analysis parameters, assumptions made, and the resulting computed responses of the slabs are provided. The analytical results are compared with that observed experimentally and general conclusions regarding the adequacy of the computed VecTor4 results are presented.

# 7.2.2.1 Modelling Approach

For the purpose of reducing computation requirements, one-quarter slab models were considered in the VecTor4 analyses of the test specimens. Sixty-five shell elements and four out-of-plane truss elements were used to represent the quarter-slab specimens and the impacting drop-weight (see Figure 7.10). A total of 298 nodes comprised the slab mesh, resulting in a model with 1,295 total degrees of freedom. The shell elements forming the slab specimens were subdivided into 25 equal thickness concrete layers, with an additional four layers used to represent the four layers of in-plane longitudinal steel present in each of the specimens. In the case of the R/FRC slabs, the end-hooked steel fibres were treated as a property of the concrete and, as such, were assumed to be uniformly distributed throughout the concrete layers of the shell elements.

To enforce symmetry, lateral and rotational restraints were provided along the slab centrelines forming the edges of the finite element model. The support reaction assemblies at the slab corners were incorporated through vertical restraint of a single node within the quarter-slab mesh, located at the centre-point of the reaction assembly. Note that because the intent of the experimental program was to provide boundary conditions which both resulted in a stable support condition but also minimized the development of axial confining stresses resulting from lateral support restraints, it was assumed that a simple roller, which permitted all rotations and translations in the plane of the slab, would be sufficient in capturing the overall slab response.



Figure 7.10 – VecTor4 Quarter-Slab Model

The rigid drop-weight was modeled using a single shell element with a high material stiffness assignment. To accommodate the flat striking face of the drop-weight used in the experimental program, the nodes comprising the rigid element were allowed to translate vertically in the global-z direction only; all rotations and in-plane translations were restrained. The drop-weight was connected to the shell elements forming the midpoint impact region of the slab using four linear elastic 'compression-only' truss elements. The massless truss elements were also assigned very large stiffness values ( $k_{truss} = 5 \times 10^6 \text{ kN/mm}$ ), creating a hard impact scenario. Because the truss elements joining the drop-weight and the slab were only capable of developing compressive stresses, no uplift impact forces were generated as a result of mass rebounding or mass bouncing.

Out-of-plane shear strength enhancements were applied to the shell elements forming, and immediately surrounding, the impact regions of the slabs using the shear strain modification approach discussed in Chapter 6. The four shell elements forming the slab impact region were the most significantly enhanced to prevent artificial punching failures from occurring directly under the drop-weight. For the single shell element forming the midpoint region of the slabs, a shear strain modification (SSM) factor of zero was considered, resulting in full suppression of the out-of-plane shear strains within the element. A SSM factor of 0.20, which represents an 80 % reduction of the out-of-plane shear strains, was considered for the remaining three elements comprising the loaded region. To enhance the shear strength of the disturbed region surrounding the loaded elements, a SSM factor of 0.50 was applied. It was assumed that the disturbed region extended over a distance of 'd' away from the edges of the loaded area, with d taken as an average value of 100 mm. Lastly, to prevent local shear failures from occurring at the idealized point-support reaction, the elements surrounding the corner support were enhanced using a SSM factor of 0.75. A plan of the VecTor4 finite element model illustrating the mesh geometry, boundary conditions, and the applied shear-strain modifications is presented in Figure 7.11.



Figure 7.11 – Quarter-Slab Finite Element Mesh (dimensions in millimetres)

The loading for each impact event was simulated by specifying an initial velocity of 8.0 m/s to four lumped masses assigned to the midside nodes of the drop-weight element. Note that because

a one-quarter slab model was considered, the lumped masses assigned to the drop-weight totaled one-quarter of the actual impact mass used in the experiments. In addition to the initial velocity assignment, a constant mass acceleration of 9.81 m/s² was also specified to drop-weight masses to simulate gravitational effects.

Because the test slabs in the experimental program were subjected to sequential impact loading, it was necessary to perform a series of analyses for each test specimen, with each analysis representing a single impact event. Upon completion of each event analysis, data reflecting the damaged slab strain state and the slab's strain history were recorded in binary format as a 'seed' file which could then be used to resume the analysis, or analyze the damaged structure under alternative loading conditions. To minimize the required computation time to perform the full series of analyses, only the initial 200 milliseconds of each impact response-time history was computed using VecTor4. It was assumed that this event duration was long enough to assess the computed post-impact response characteristics of the slabs (i.e., damping behaviours and residual displacements) and that only minimal, if any, slab damage would occur as a result of additional low amplitude response oscillations which were typically estimated to occur at the ends of the 200 millisecond events. Additionally, secondary impacts due to mass rebounding which occurred after the initial 200 milliseconds of the impact event have been neglected.

Prior to performing the dynamic analyses on the damaged slabs, intermediate analyses were first required to return the drop-weight from its rebounding state, back to its initial starting position. To do this, a monotonic analysis consisting of only one load stage with a 10 Newton point load was applied to the drop-weight. This small load level was enough to position the drop-weight in contact with the slab without causing any significant additional slab displacements. Additionally, the performance of the intermediate monotonic analysis also resets all motion parameters (nodal velocities, nodal accelerations) from the end of the previous impact analysis, back to zero.

The staged analysis procedure used for each slab specimen was performed as follows:

- i. Perform dynamic analysis (200 ms event) of initial impact test on undamaged slab; output seed file reflecting damaged slab strain state.
- ii. Upload seed file; perform monotonic analysis (single load stage) to reposition dropweight; output updated seed file.

iii. Update impact mass level; upload updated seed file; perform dynamic analysis of next impact event.

For each slab, steps (ii) and (iii) were repeated until the full series of sequential impact events performed in the laboratory was completed, the computed response of the slab had indicated that the slab had failed, or the VecTor4 analysis became unstable.

## Behavioural Models and Analysis Parameters

As discussed in the previous chapter, a set of default material and structure behavioural models has been defined for VecTor4, and these default options were used exclusively for the verification analyses performed in this thesis. However, for the dynamic analyses of the slabs under impact, one non-default material model was considered.

By default, VecTor4 considers the formulation proposed by Seckin (1981) with incorporation of the Bauschinger effect to model the hysteretic behaviour of steel reinforcing bars. Finite element modelling of the Saatci impact beams (Saatci, 2007; Guner, 2008), showed that the Bauschinger effect considered in the Seckin hysteretic model had the tendency to recover plastic strains in the yielded longitudinal bars as a result of low-amplitude post-impact vibrations. Analytically this strain recovery was apparent from the recovery of residual displacements in the computed displacement-time history responses of the beams, and differed from that observed experimentally. To prevent artificial plastic strain recovery from occurring in the VecTor4 analyses of the impact slabs, an alternative reinforcement hysteresis model was selected.

An elastic-plastic model with curvilinear hardening was selected for the analysis of the impact slabs. Unlike the implemented Seckin model, the Bauschinger effect is not considered in the hysteretic response of this constitutive model and, as such, this model does not recover plastic strains in yielded reinforcement as a result of low-amplitude cycling. Details regarding the formulation of the reinforcement model are provided by Wong et al. (2012).

To illustrate the plastic strain recovery described above, VecTor4 was used to perform a simple cyclic analysis on a reinforced concrete axial member. The member was subjected to a cyclic axial strain protocol which resulted in extensive reinforcement yielding within the first cycle, and

followed with low-amplitude strain cycling thereafter. The analysis was performed using the default and the non-default steel hysteresis models, and the results are summarized in Figure 7.12. In comparing the computed reinforcement responses it can be seen that the elastic-hardening model behaves elastically under post-yield low-amplitude cycling. However, the Seckin model recovers more than 20 % of the initially developed plastic strain as a result of the low-amplitude cycling. The plastic reinforcement strain histories from both analyses are summarized in Figure 7.12b.



Figure 7.12 - VecTor4 Reinforcement Response under Post-Yield Low-Amplitude Cycling

As was the case in the monotonic verification studies, the direct tensile strengths of the slabs were approximated as  $0.33\sqrt{f_c}$ , the slab densities were taken as 2,400 kg/m³, and Popovics HSC model was selected as the compression base curve for the 50 MPa nominal strength concretes.

To reduce computation time, less stringent convergence criteria were used in the dynamic analyses. The specified convergence tolerance was taken as 1.000050 and was computed on the basis of nodal displacements (translations and rotations). Under-relaxation was applied using a constant averaging factor of 0.50, which averaged the local strain values from one iteration to the next. Lastly, a maximum of 30 iterations were performed within each time step of the analyses.

#### 7.2.2.2 Monotonic Behaviour

In addition to the dynamic analyses of the impact slabs, an investigation regarding the analytical monotonic behaviours of the specimens was performed. Results from the monotonic analyses were used primarily to aid in the design of the experimental program, but were also useful for estimating the behaviours of the slabs under more conventional loading conditions.

The same finite element mesh designed for the impact analyses was used for the monotonic investigation. The analyses were performed in a displacement-controlled manner, with the controlling displacements applied within the impact area of the quarter-slab model. Analyses pertaining to slabs TH2 through TH7 were performed and the computed load-deflection responses are presented in Figure 7.13.



Figure 7.13 - Computed Monotonic Responses for Test Slabs

Figure 7.13a illustrates the influence of the longitudinal reinforcement ratio on the analytical load-deflection responses of the slabs. As all of the slabs in the experimental program possessed relatively low reinforcement ratios, flexure dominated behaviours were estimated to primarily control the behaviours of the slab specimens. Increased reinforcement levels resulted in increased computed capacities with only marginal reductions in the computed ductilities. In all cases, prior to reaching peak load resistances, the longitudinal reinforcing bars comprising the bottom mat of reinforcement were estimated to undergo tensile strains in excess of 5 % throughout the midpoint regions of the slabs. The governing failure modes of the RC slabs were attributed to extensive tensile steel yielding following by flexural concrete crushing along the slabs' centrelines forming the interior edges of the quarter-slab model.

The influence of the steel fibre volume fraction is illustrated in Figure 7.13b. In contrast to the results obtained for the nonfibrous RC slabs, the addition of the steel fibres was estimated to only minimally impact the ultimate capacities of the slabs, with results suggesting that the steel fibres primarily enhanced the tension-stiffening behaviours. As the steel fibre volume fraction was increased, the load required to yield the R/FRC slabs was estimated to increase, and the ductilities of the flexure-controlled slabs were found to decrease. R/FRC slabs TH3 and TH4 which were comprised of FRC with fibre volume fractions of 0.50 % and 1.00 %, respectively, were estimated to have capacities and ductilities which were similar to that calculated for RC slab TH2. The computed response of R/FRC slab TH5 which contained 1.50 % fibre volume fraction was significantly less ductile than that of the RC slabs and the R/FRC slabs containing lower fibre volumes.

In addition to verifying the functionality of the slab FE models developed, the analytical responses presented in Figure 7.13 were useful in providing several general observations regarding the monotonic behaviours of the slabs forming the experimental program:

- The RC and R/FRC slabs comprising the experimental program were expected to be flexure controlled with extensive steel yielding followed by concrete crushing.
- The RC slabs exhibited highly ductile behaviours.

- Increased longitudinal reinforcement ratios resulted in increased load capacities and marginal reductions in the computed ductilities.
- The addition of steel fibres was expected to enhance the tension-stiffening behaviours.
- The computed ductilities of the flexure-controlled R/FRC slabs decreased as the steel fibre volume fractions increased. However, slabs with 0.50 % and 1.00 % steel fibre volume fractions were estimated to achieve similar ductility levels as RC slab TH2.
- The addition of steel fibres was found have little influence on the computed capacities of the flexure-controlled slabs.

It is worth noting that previous experimental studies pertaining to R/FRC beams have shown similar tendencies with the analytical results reported in this investigation. Aoude et al. (2012) found that the addition of relatively small amounts of end-hooked steel fibres (1.00 % or less in their testing program) had no significant effect on the resulting load capacities of flexure-controlled beams. An experimental program performed by Cucchiara et al. (2004) obtained similar results when a fibre volume fraction of 1.00 % was added to a flexure-controlled beam, and found that the addition of 2.00 % steel fibres led to marginal increases in the flexural capacity of the beam and resulted in a reduction of the beam's ductility.

## 7.2.2.3 Selection of Dynamic Analysis Parameters

This subsection presents the results from a series of brief parametric investigations which were used to select appropriate analysis parameters for the VecTor4 impact slab analyses. The required dynamic analysis parameters include the time step used in the numerical time integration scheme, the amount of supplemental viscous damping used to enhance solution stability, and the use of material strain rate enhancement models.

## Time-Step

The size of the time step used in time-increment based dynamic analyses can significantly influence the resulting computed responses. In general, as the size of the time step is reduced, errors attributed to the time integration scheme are also reduced. The impact analyses of the test slabs were performed using Newmark's average acceleration method which, as previously noted, is a desirable method for nonlinear MDOF applications as it is unconditionally stable and possesses no inherent numerical damping. Ideally when using Newmark's average acceleration

method, a time step which is at least one order of magnitude smaller than the period of the highest contributing modal frequency should be selected. However, knowing in advance which modes contribute significantly to the overall response is nearly impossible, particularly in the case of nonlinear MDOF systems. Additionally, computation intensive or iterative solution algorithms may make the use of very small time steps impractical for the analysis of structures possessing many dynamic degrees of freedom. As such, both solution accuracy and computation time should be considered when selecting an appropriate time step for dynamic analyses using VecTor4.

The selection of the time step used in the analyses of the slab impact tests was based on a limited investigation of the analytical response of event TH2-1. The analysis case of an initial impact event on an undamaged slab was chosen as it represents an analysis scenario which was likely to require the smallest time step size to achieve reasonable levels of accuracy. Analyses were performed using time steps ranging from 0.001 seconds to 0.00001 seconds, and all other parameters were kept constant.

The computed midpoint displacement-time histories from five analyses performed using different time steps are presented in Figure 7.14a. The use of the 0.001 second time step, which was the largest step size considered, resulted in a highly unstable displacement response. Additionally, the displacement amplitude and the period of the midpoint displacement response computed using the large time step were significantly greater than those calculated using smaller step sizes. Results obtained using the 0.0005 second time step were stable throughout the analysis; however, they still differed significantly from those computed using smaller steps. Lastly, the results obtained using time steps ranging from 0.0001 seconds to 0.00001 seconds exhibited higher levels of agreement amongst each other, with significantly less variation in period and displacement amplitude.

For clarity, the computed displacement-time and reaction-time histories from only the three analyses performed using time steps of 0.0001 seconds or smaller are presented in Figure 7.14b and Figure 7.14c. It can be seen that only small variations amongst the three computed results are noticeable within the first two full cycles of the responses. However, in the later stages of the analysis, discrepancies in the displacement amplitude and period are evident, with results from

the analyses performed using larger time steps exhibiting period elongation. The extent of the period elongation and the reduction in displacement amplitude reduction correlates with the increasing time step size.



Figure 7.14 – Influence of Time Step, Slab TH2 ( $\rho_l = 0.420$  %;  $V_f = 0$ )

The computation demands resulting from the different time step sizes are summarized in Figure 7.15. Evident from the figure, significant computation costs are incurred as a result of decreasing the time step size. As such, it is necessary to select a time step which balances both solution accuracy and computation demands. Considering this objective, a time step of 0.0001 seconds was selected for the performance of all of the slab impact analyses. The use of this time step for the analysis of impact event TH2-1 resulted in reasonable solution accuracy, achieving near

perfect agreement in the initial phase of the response when compared to that obtained using smaller step sizes. Additionally, the analysis run-time time was found to be significantly shorter than that required for the two analyses performed using smaller step sizes. Lastly, it should be reiterated that the investigation regarding time step requirements was performed considering an undamaged slab. Subsequent analyses considering impact events performed on damaged slabs are likely to result in higher levels of numerical accuracy as a result of reduced frequency contents.



Figure 7.15 – Influence of Time Step Length on Analysis Run-Time

### Supplemental Damping

To enhance solution stability, low levels of supplemental damping were considered in all of the slab impact analyses undertaken. The required damping was estimated from multiple analyses of one impact event (event TH2-1) which were performed using a range of damping levels. The presented damping investigation was not intended to be exhaustive, rather was used to develop a simple estimate of supplemental damping which could be used for all of the slab analyses.

Rayleigh damping, which is comprised of both mass and stiffness proportional contributions, is commonly used to simulate the damping behaviours of structures as it has shown reasonable agreement with data from experiments (Chopra, 2012). However, in the case where viscous damping is used primarily to stabilize the solution algorithm, as is the case in this study, stiffness proportional damping is perhaps advantageous.

- As discussed in Section 7.1.2.2, damping levels can significantly change throughout the course of a nonlinear analysis. Since the proportional damping coefficients ( $a_0$  and  $a_1$ ) are only calculated at the beginning of the VecTor4 analysis, the inclusion of mass proportional damping contributions can lead to excessive damping as the structure stiffness decreases.
- The development of artificial high frequency modes is inherent in the finite element method.
   Stiffness proportional damping most significantly impacts the highest frequency modes of the structure.

Ultimately, stiffness proportional damping was considered as it ensured that, as the stiffness of the slabs decreased due to the development of damage and plastic strains, the level of applied damping decreased.

The displacement-time histories for impact event TH2-1 pertaining to five different damping levels are presented in Figure 7.16. The selected stiffness proportional damping coefficient,  $a_1$ , ranged from 0.06 x 10⁻⁴ s to 0.62 x 10⁻⁴ s, resulting in damping ratios for the first mode of the undamaged slab which ranged from 0.10 % to 1.25 %, respectively. The damping matrix of the slab was calculated using:

$$[c] = a_1[k] \tag{7-47}$$

It should again be noted that the computed modal damping ratios decreased over the course of the analyses and, as such, do not reflect the amount of damping applied. The selected stiffness proportional damping coefficients, however, were constant throughout.

From Figure 7.16, it can be seen that as the amount of the supplementary damping increased, the peak displacement amplitude within the first half-cycle decreased. Additionally, it can be seen that the period of the response decreased as a result of the increased damping levels.

For clarity, the computed displacement and support reaction responses from the five damping levels considered are presented individually in Figure 7.17. The analysis performed with the lowest amount of damping ( $a_1 = 0.06 \times 10^{-4}$  s) was highly unstable during the initial mass impact phase of the analysis and resulted in a localized shear failure of the slab. The localized shear
failure led to significantly larger displacement amplitude and an increased response period. The analysis performed with the second lowest amount of damping ( $a_1 = 0.16 \times 10^{-4} \text{ s}$ ) was adequately damped throughout the majority of the analysis; however, it exhibited signs of instability in the form of poor convergence and irregular displacement and reaction responses during the initial mass impact phase of the analysis. The remaining three damping levels ( $a_1 = 0.32 \times 10^{-4} \text{ s}$  to  $0.62 \times 10^{-4} \text{ s}$ ) all resulted in stable responses throughout the analyses.



From the support reaction-time histories presented in Figure 7.17 it is evident that high frequency contributions have been suppressed as a result of the increased damping levels; however, in the case of  $a_1 = 0.32 \times 10^{-4}$  s, the overall reaction response seems to be retained when compared to that computed with less damping.

Based on the results presented above, stiffness proportional damping with a coefficient of  $a_1 = 0.32 \times 10^{-4}$  s was selected for the analyses of all slabs and all impact events. Proportional damping in the range of 0.16 x  $10^{-4}$  s  $< a_1 < 0.32 \times 10^{-4}$  s may have also been adequate. However, because the required stiffness damping coefficient was estimated from the analysis of only one impact event for one slab, the value of  $a_1 = 0.32 \times 10^{-4}$  s was selected to ensure that reasonable solution stability was obtained for all analysis cases.



Figure 7.17 – Results from Damping Investigation

## Strain Rate Effects

As noted previously, dynamic increase factors (DIF) are considered by default in VecTor4 and are used to account for the material strain rate effects. The default DIF-strain rate relationships for concrete are based on the provisions of the *fib* Model Code (2010), and those for steel reinforcing bars are in accordance with the model proposed by Malvar and Crawford (1998) (see Section 7.1.3). To assess the influence of the dynamic increase factors on the computed responses of the test slabs, analyses were performed with and without strain rate effects considered. In addition to the default option, an alternative strain rate model for steel reinforcing bars based on the provisions of a CEB-FIP synthesis report (CEB, 1988) was also considered to assess whether the default rate effect models are appropriate for the analysis of the impact slabs.

Figure 7.18 presents the analytical results against that measured experimentally for impact event TH2-1. It can be seen that the analyses considering strain rate effects resulted in significantly stiffer responses. The peak midpoint displacement, the period of the response, and the residual midpoint displacement are all reduced as a result of strain rate enhancement. The support reactions, however, are only marginally affected by the inclusion of rate effects. The calculated peak reaction forces were nearly identical in all cases, and differences attributed to the inclusion of rate effects were essentially limited to the response period.



Figure 7.18 - Influence of Strain Rate Effects on Computed Response; Event TH2-1

When compared to the experimental response-time histories, the analyses performed considering rate effects provided much better estimates of the peak midpoint displacement. For the impact

event considered, the period of the response seemed to be slightly underestimated when rate effects were considered and slightly overestimated when they were neglected. Additionally, all of the analyses underestimated the residual midpoint displacement of the slab; however, the analysis performed without rate effects provided a better estimate as a result of additional damage incurred during the initial impact phase of the analysis. All analyses underestimated the peak reaction forces measured during the experiment and all of the analyses resulted in similar levels of accuracy with respect to the overall support reaction-time history response.

Minor differences can be observed amongst the responses obtained using the Malvar-Crawford model and the CEB-FIP model to account for reinforcement strain rate effects. In comparison to the response calculated using the default model, the CEB-FIP relationships resulted in a slightly larger peak displacement and response period; the calculated residual displacement was essentially the same. With the exception of minor differences in the period of the support reaction response, the computed reactions were similar using both reinforcement strain rate models.

The computed response of the slab was found to be significantly impacted by the inclusion of strain rate effects, with the general tendency being that strain rate enhancement resulted in stiffer responses. Similar responses were computed using different strain rate models for the reinforcement, suggesting that for this specific analysis either of the two models could be considered. However, for the purpose of being consistent with other sectional analysis based VecTor software programs (Guner and Vecchio, 2012), the Malvar-Crawford reinforcement strain rate model has been considered in the analysis of the impact slabs.

## 7.2.2.4 Influence of Modelling Assumptions

A number of assumptions were considered in the development of the VecTor4 finite element mesh used for the analyses of the impact slabs. The following subsection presents the methodology used in defining the mesh, and presents additional discussions pertaining to the most significant modelling decisions: using a quarter-slab idealization to model the behaviours of the slabs under impact, and artificially enhancing the out-of-plane shear strengths of the slabs to accommodate disturbed regions. The discussions are supported by supplementary investigations.

#### Quarter-Slab Idealization

To reduce the required computation time of the dynamic impact analyses, a finite element mesh representing only one-quarter of the test slabs was considered. It was assumed that the simplification of the non-symmetric boundary conditions used in the experimental program would have little effect on the overall behaviours of the slabs. To verify this assumption, in addition to the analysis performed using the idealized quarter-slab model, impact event TH2-1 was also analyzed using a finite element mesh comprising the full-slab specimen (see Figure 7.19). The boundary conditions considered in the full-slab model were specified to match the lateral restraint conditions provided in the laboratory. The mesh density, the behavioural models, and the selected dynamic analysis parameters (e.g., the time-step and the damping parameters) were the same as those used for the quarter-slab analyses.



The computed midpoint displacement- and the support reaction-time histories from the two models are presented in Figure 7.20. The response computed from the quarter-slab model closely matched that computed using the full-slab model, in all respects. Minor differences between the computed displacement amplitudes were found; however, the small level of deviation was

considered to be negligible. The analytical response frequencies and the computed damping behaviours from the two models were also found to be in good agreement.



The vertical support reaction-time history for each of the corner supports comprising the full slab model are presented in Figure 7.21. It can be seen that although the total support reaction-time history computed from the full-slab analysis closely matched the results from the quarter-slab model, the computed vertical reaction-time histories did slightly fluctuate amongst the four corner supports. However, in general, the computed vertical corner restraints were similar and no additional significant contributions to the overall slab response were captured as a result of modelling the full-slab.



Figure 7.21 – Variation of Calculated Support Reactions; Full-Slab Model

## Shear Strength Enhancement

In developing the finite element mesh used for the analysis of the impact slabs, it was assumed that as a result of local disturbances, the behaviours of the slabs immediately surrounding the mass impact area did not conform to the governing plane-section theory considered in VecTor4. Contributions from local confinement, direct strut action, and inclined crack propagation resulting from punching are not directly considered in the VecTor4 sectional analysis algorithm. To accommodate these disturbances in an approximate sense, the out-of-plane shear strains were suppressed throughout the disturbed region using the methodology discussed in Chapter 6.

Out-of-plane shear strength enhancement, in the form of shear strain suppression, was applied over a region which extended d away from the edges of the impacting mass. The boundary of the disturbed region was based on the assumption of a 45° shear cone which extended from the edge of the impacting mass to the bottom mats of reinforcement. Many design guidelines require that punching shear capacities be evaluated considering a critical shear perimeter which is located at a distance d/2 away from the edge of the column or concentrated load (ACI 318 2008; CSA-A23.3 2004, *fib* MC 2010). However, the application of such provisions combined with nonlinear sectional analysis tools and for purposes of assessment as opposed to design is somewhat unfounded.

To investigate the sensitivity of the analytical results to the above assumptions regarding out-ofplane shear strength enhancement, additional dynamic and static analyses were performed using alternative disturbed region boundary dimensions. In total, three cases were considered: 1) the boundary of the disturbed region was defined at a distance *d* away from the edge of the impact mass/loading plate, 2) the region was defined at a distance of *d*/2 away from the edge of the impact mass/loading plate, and 3) the disturbed region was not considered. In cases 1) and 2) the out-of-plane shear strains were reduced by 50 % within the disturbed regions. An illustration of the three finite element meshes used for the investigation is presented in Figure 7.22.

The results from the static analyses of RC slabs TH2, TH6, and TH7 are presented in Figure 7.23. For each RC slab, analytical responses were computed using each of the three meshes presented above in Figure 7.22. From the computed responses it can be seen that the out-of-plane shear strength enhancement significantly increased the capacities and the ductilities of the three

RC slabs. As the size of the disturbed region was increased from that defined by distance d/2 to that defined by d, additional increases of approximately 10 % of the slabs' capacities were attained. Furthermore, increasing the size of the disturbed region also changed the failure modes of the slabs. With a disturbed region defined by d/2 failure was ultimately governed by punching outside of the disturbed region. When the size of the disturbed region was further increased to a region defined by d, the failure modes of the slabs were controlled by extensive concrete crushing within the midpoint region of the slab, which ultimately propagated failure within the disturbed region.



Figure 7.22 – FE Meshes Considered in Disturbed Region Investigation



Figure 7.23 – Influence of Shear Strength Enhancement on Computed Static Response

Figure 7.24 presents the displacement-time history results from a series of dynamic analyses performed for RC slab TH2. Analyses were carried out for the full series of sequential impact events performed on slab TH2 in the laboratory for each of the three finite element meshes presented above in Figure 7.22.

It can be seen that when the disturbed region was neglected entirely, the displacement results from the first impact event are highly unstable and significantly overestimate the midpoint displacement amplitude recorded experimentally. Additionally, the slab is estimated to abruptly fail as a result of localized punching in the analysis of the second impact event. Analysis of the third impact event could not be performed.



Figure 7.24 – Influence of Shear Strength Enhancement on Computed Dynamic Response

In the cases of the analyses performed using meshes with shear strength enhancement, it can be seen that the computed responses correlate much better with that observed experimentally. In the analysis of the first impact event, little difference can be seen amongst the responses computed using the two disturbed region sizes (i.e., d and d/2). However, in the analyses of subsequent impact events TH2-2 and TH2-3, the mesh considering the disturbed region defined by d/2 overestimated the damage attributed to punching and, as a result, became unstable prior to completing the analysis of the third impact event.

From the results presented within this section, it can be seen that the use of out-of-plane shear strength enhancement within the disturbed region of the slab surrounding the impact area significantly improved the computed displacement-time histories resulting from the dynamic analyses. For the single test specimen considered in the investigation (RC slab TH2), the highest level of agreement with the experimental results were achieved when a disturbed region defined by a boundary located at distance d away from the edge of the impacting mass was modelled. As was previously discussed in Chapter 6, and perhaps of more significance, other researchers employing layered sectional algorithms with two-dimensional analysis tools have also considered disturbed regions which are located in the order of d from the loading/support locations, and have obtained reasonable agreement with experimental data (Bentz, 2000; Guner, 2008). As such, the sizing of the disturbed region as illustrated in the finite element mesh presented in Figure 7.22a was used explicitly for the analyses of all RC and R/FRC slabs, and all impact events.

# 7.2.2.5 Analytical Responses of RC Slabs

This subsection of the thesis presents the computed responses of the non-fibrous RC slabs: slabs TH2, TH6, and TH7. Pilot slab TH1 was not analyzed as several of the displacement measurements were compromised during the initial pilot testing.

For comparison, the analytical midpoint displacement-time, total support reaction-time, and impulse-time histories have been plotted alongside the experimental results for each impact event. Additional discussions pertaining to the computed slab deformation behaviours and reinforcing bar strain-time histories are provided for selected impact events.

#### *Slab TH2* ( $\rho_l = 0.420$ %; $V_f = 0$ )

General response-time histories from the analyses of the three impacts performed on slab TH2 are presented in Figure 7.25. From the displacement-time histories presented it is evident that VecTor4 accurately estimated the peak midpoint displacements in all cases. Slab damage, in the form of residual displacement, was not captured well in the analysis of the initial impact performed on the undamaged slab (i.e., impact event TH2-1). However, the analysis of subsequent impact events considering the already damaged slab provided good estimates of the displacement rebound amplitudes and of the residual midpoint displacements.



Figure 7.25 – Analytical Response-Time Histories; Slab TH2 ( $\rho_l = 0.420$  %;  $V_f = 0$ )

Comparing the computed support reaction-time histories to those measured experimentally, it can be seen that the peak reaction forces were significantly underestimated in the three impact events; however, the general shapes of the computed reaction-time responses seem to be in agreement with the experimental data. Examination of the calculated support reaction impulses suggest that although VecTor4 underestimated the peak reactions in all cases, the computed impulses agree well with the experimental data and had the tendency to somewhat overestimate the peak impulse within the first cycle of the response.

## *Slab TH6* ( $\rho_l = 0.29$ %; $V_f = 0$ )

Similar levels of accuracy were achieved in the analyses of the slab TH6 (see Figure 7.26). Accurate estimates of the peak midpoint displacements were obtained for all impact events, and residual displacements were computed poorly in the analysis of the undamaged slab but reasonably well in the analysis of the second impact. Peak reactions were underestimated; however, the computed reaction impulses within the first cycle of the responses suggest that the overall reaction responses were, on average, captured reasonably well.



Figure 7.26 – Analytical Response-Time Histories; Slab TH6 ( $\rho_l = 0.273$  %;  $V_f = 0$ )

# *Slab TH7* ( $\rho_l = 0.592$ %; $V_f = 0$ )

The analytical results pertaining to slab TH7 are presented in Figure 7.27. The results from the first two impact events generally overestimated the post-impact stiffnesses of the slab. As a result, the analytical response periods were shorter and the residual displacements were smaller than those observed experimentally. The peak displacements were computed with high accuracy; however, under the third impact event in which punching behaviour was prominent, the overall midpoint displacement behaviour was somewhat underestimated.



Figure 7.27 – Analytical Response-Time Histories; Slab TH7 ( $\rho_l = 0.592$  %;  $V_f = 0$ )

In comparison to the analytical results for slabs TH2 and TH6, the support reaction-time histories for slab TH7 were computed with slightly improved accuracy; however, the peak reactions were still greatly underestimated in two of the three events. The shapes of the support reaction-time histories matched the experimental results quite well, particularly in terms of the timing of initial and even secondary peaks within the support reaction responses. Good agreement was obtained between the experimental and the analytical impulse responses, with VecTor4 having the tendency to overestimate the peak impulse values within the first cycle of the responses.

A summary of the calculated midpoint displacements for the RC slabs is presented is Table 7.4. From the tabulated data it can be seen that on average, high accuracy estimates of the peak midpoint displacement values were obtained with a mean analytical-to-experimental ratio of 1.02 and a coefficient of variation of 9.1 %. Estimates of the residual displacements, however, were not computed with the same level of accuracy. The mean analytical-to-experimental ratio for the residual displacements was found to be 0.50 with a coefficient of variation of 68 % which reflects the relatively low level of agreement with the experiments. For all of the RC slabs, the least accurate estimates of the residual displacements were obtained from the analyses of the initial impact events.

	Peak Displacement (mm)			Residual Displacement (mm)			
Event	Test	VT4	VT4/Test	Test	VT4	VT4/Test	error
TH2-1	13.2	14.0	1.06	2.7	0.3	0.12	2.4
TH2-2	18.7	21.5	1.15	3.0	2.2	0.71	0.9
TH2-3	26.6	29.1	1.10	8.9	9.6	1.08	0.7
TH6-1	18.3	17.2	0.94	 6.2	1.7	0.28	4.5
TH6-2	27.7	29.6	1.07	11.4	10.3	0.90	1.1
TH7-1	12.8	13.1	1.02	 2.6	0.3	0.12	2.3
TH7-2	18.3	18.5	1.01	3.3	0.9	0.28	2.3
TH7-3	28.5	23.7	0.83	11.5	5.7	0.49	5.8
		Mean =	1.02		Mean =	0.50	2.5 mm
		C.O.V. =	9.1 %	C.O.V	$V.(\sigma_{res}^*) =$	68 %	(1.7 mm)

Table 7.4 – Summary of Midpoint Displacement Results for RC Slabs

* standard deviation of absolute residual displacement error

It should be noted that the computation of residual displacements poses a significant challenge as they rely heavily on accurate modelling of several complex behavioural mechanisms: localized material damage, post-peak material responses, and plastic strain development of the materials, all of which are difficult to estimate in specimens subjected to well-defined loading conditions. Furthermore, the current VecTor4 modelling procedure only considers steel plastic strain development from average reinforcing bar yielding. Plastic reinforcing bar strains developed across the crack locations are not retained, which may also play a role in underestimating the residual slab displacement. Lastly, some consideration should be given to the magnitudes of the slabs' residual displacements, particularly those corresponding to the initial impacts. If the analytical estimates are assessed in the form of absolute error, the mean residual displacement error is calculated to be 2.5 mm, with a standard deviation of approximately 1.7 mm. This level of error is certainly appreciable; however, in the overall context of the problem, it is perhaps not of significant concern.

The measured and computed peak reactions are presented in Table 7.5. It can be seen that the initial peak values of the support reaction forces were consistently underestimated, with the VecTor4 results being in the order of 50 to 60 % of that measured experimentally. However, not indicative from the table, the analytical reaction force behaviours were computed reasonably well beyond the first half-cycles of the responses.

	Peak Su	pport Reacti	ion (kN)	Peal	Peak Impulse [*] (kN-s)			
Event	Test	VT4	VT4/Test	Test	VT4	VT4/Test		
TH2-1	556	331	0.59	2.24	2.55	1.14		
TH2-2	676	353	0.52	2.83	3.08	1.09		
TH2-3	580	307	0.53	3.34	3.32	0.99		
TH6-1	483	256	0.53	2.2	2.47	1.12		
TH6-2	458	234	0.51	2.86	2.74	0.96		
TH <b>7-</b> 1	605	385	0.64	2.25	2.55	1.13		
TH7-2	588	441	0.75	3.01	3.02	1.00		
TH7-3	368	392	1.07	3.25	3.42	1.05		
		Mean =	0.64		Mean =	1.06		
		C.O.V. =	28 %		C.O.V. =	6.2 %		

Table 7.5 – Summary of Support Reaction Results for RC Slabs

* maximum value within first cycle of response

In contrast to the computed reaction forces, the analytical support reaction impulses were in much better agreement with the experimental responses. In comparing the peak impulse values pertaining to the first cycle of the reaction-time history response, it can be seen that VecTor4 had the tendency to overestimate the peak values. However, in general, support reaction impulses were computed with good agreement, resulting in a mean analytical-to-experimental ratio of 1.06 with a coefficient of variation of 6.2 %.

# Displaced Shapes and Failure Modes

In addition to validating the computed deformations of the RC slabs, analytical displacement profiles were used to assess if, and to what degree, punching shear failures were estimated to occur. Displacement profiles from the three impacts performed on slab TH2 are presented in Figure 7.28b. The displaced shapes shown represent the slab deformations at the time of the peak computed midpoint displacement, and have been plotted against the peak experimental displacement profiles.



Figure 7.28 – Analytical Displacement Profile; Slab TH2 ( $\rho_l = 0.420$  %;  $V_f = 0$ )

It can be seen that the computed displacement profiles of slab TH2 are in good agreement with the experimental results. VecTor4 estimated the initial development of localized deformations under the second impact as a result of punching, and in the analysis of the third impact event (event TH2-3) the punching attributed failure of the slab was accurately captured. The analytical slab displacements across the section closely matched the experimental data in all of the analyses. It should be noted that with regard to the measured peak displacement profile corresponding to the third impact, the experimental punching region was measured on the bottom

surface of the slab. As such, the length of the punched region measured experimentally was found to be larger than that computed using VecTor4 which computes displacements with respect to the mid-height of the slab.

The analytical results obtained for TH6 were found to agree well the experimental displaced shapes of the slab (see Figure 7.29a). In the case of slab TH7, punching was accurately estimated to occur under the third and final impact; however, the deformed shape of the punched slab was computed with less accuracy. Additionally, the displacement profiles corresponding to the second and third impact events were generally underestimated.



The deformed shapes of the RC slabs were estimated well and the punching failures observed in the experiments were captured in the analyses. In some cases, particularly for slab TH7, the accuracy of the computed displaced shapes tended to deteriorate over the course of the sequential analysis protocol. However, some deterioration of the analytical results in the performance of sequential event analyses is to be expected as errors in the analytical results will accumulate from the analysis of one impact event to the next.

#### Reinforcing Bar Strains

Lastly, the computed reinforcing bar strain-time histories from selected impact events of the RC slabs are compared with experimental strain measurements. Note that the analytical results

represent average reinforcing bar strain values. Strain gauge S1, which is located in the midpoint region of the slab on the interior bottom-face mat of reinforcement, was considered.

In general, it was found that that the accuracy of the computed bar strains was somewhat inconsistent, and in many cases the analytical strains differed significantly from the experimental values. As was previously noted, residual strain errors may possibly be attributed to neglecting local plastic strains across the cracks in the analytical procedure. In the initial two impact analyses pertaining to slab TH2, the calculated peak bar strains and the initial shape of the strain-time history response closely matched the experimental data (see Figure 7.30). However, the post-peak responses were computed with less accuracy, with the general tendency being underestimations of the residual strains and overestimations of the response amplitudes. From Figure 7.30c, it can be seen that the analysis of the third and final impact of slab TH2 resulted in significant overestimation of the both the peak strain level and the residual bar strains.



Figure 7.30 – Analytical Reinforcing Bar Strain-Time Histories; Slab TH2 ( $\rho_l = 0.420$  %;  $V_f = 0$ )

In the case of slab TH6, a test slab in which the resulting damage from the impacts was significantly more apparent than that observed for TH2, overestimations in the peak strains were computed in the analyses of both impact events (see Figure 7.31). The computed bar strain-time histories for the slab did not exhibit the rapid post-peak decay measured in the experiments, resulting in significant overestimations of strain amplitude throughout the analytical responses.



Figure 7.31 – Analytical Reinforcing Bar Strain-Time Histories; Slab TH6 ( $\rho_l = 0.273$  %;  $V_f = 0$ )

## 7.2.2.6 Analytical Responses of R/FRC Slabs

This section presents the computed responses for the R/FRC slabs: TH3, TH4, TH5, and TH8. The analytical midpoint displacement-time, the support reaction-time, and the impulse-time histories have been presented for each impact event analyzed. Additionally, discussions pertaining to the validity of the computed displaced shapes of the slabs and to the calculated reinforcing bar strain-time histories are provided.

## *Slab TH3* ( $\rho_l = 0.420$ %; $V_f = 0.50$ %)

The analytical results from the five impact tests performed on slab TH3 are presented in Figure 7.32. Note that excessive data filter settings used during the testing of the impact TH3-1 rendered the experimental data unusable, and as such, comparisons between the analytical and experimental responses have not been included for TH3-1. However, to be consistent with the loading protocol used in the laboratory, the analysis of the first impact event was performed, and the VecTor4 results have been included in the figure.

It can be seen that the analytical results from events TH3-2 through TH3-4 achieved similar levels of accuracy as that attained in analyses of the RC slabs (see Section 7.2.2.5). In general, the computed midpoint displacements were estimated well in the initial impact tests, and the overall shape of the support reaction-time histories agree with the experimental data. In all cases, the peak reaction force was significantly underestimated with analytical values being in the order of 50 % to 60 % of the experimentally measured forces. However, the values of the peak impulses within the first cycles of the responses were estimated more accurately.



Figure 7.32 – Analytical Response-Time Histories; Slab TH3 ( $\rho_l = 0.420$  %;  $V_f = 0.50$  %)



From Figure 7.32e it can be seen that the analytical results from the analysis of the fifth and final impact test were found to significantly differ from those measured experimentally. The value of the peak midpoint displacement was calculated with reasonable accuracy; however, the overall shape of the computed displacement response possessed significant errors in the form of a larger period and rapid damping, suggesting that the post-impact stiffness was severely underestimated in the analysis. Additionally, the computed residual displacement of the highly-damaged slab seemed to recover in a manner that was not observed experimentally.

### *Slab TH4* ( $\rho_l = 0.420$ %; $V_f = 1.00$ %)

The analyses of impact tests performed on slab TH4 are plotted alongside the experimental responses in Figure 7.33. Throughout the sequence of analyses the computed responses remained stable and provided reasonable estimates of the experimental behaviours. Analyses of the first five impacts (events TH4-1 through TH4-5) resulted in high accuracy estimates of the midpoint displacements and damping characteristics, and reasonable estimates of the response periods. The computed support reaction-time histories of the initial impacts agreed reasonably well with the experimental responses, but reduced accuracies were attained through the progression of the sequential impact protocol. In all cases the peak reactions were underestimated; however, the amplitudes of the post-impact responses (i.e., after the first half-cycle) were computed well.



Figure 7.33 – Analytical Response-Time Histories; Slab TH4 ( $\rho_l = 0.420$  %;  $V_f = 1.00$  %)



Slab TH4 ( $\rho_l = 0.420$  %;  $V_f = 1.00$  %)

The analyses of the sixth and seventh impacts provided less accurate estimates of the experimental responses: the peak midpoint displacements were slightly overestimated, the periods of the computed response periods were overestimated, and the general shapes of the support reaction responses showed little agreement with the high frequency responses measured experimentally. Similar to the analytical results pertaining to R/FRC slab TH3, the overall stiffness of slab TH4 was underestimated in the final impacts; however, most of the peak displacements were recovered resulting in underestimation of the residual displacements.

## *Slab TH5* ( $\rho_l = 0.420$ %; $V_f = 1.50$ %)

In total, ten impact tests were performed on slab TH5. However, due to noted computational instabilities, analyses of only the first eight tests were performed. The results from the analyses of slab TH5 are presented in Figure 7.34. In comparing the experimental and the analytical midpoint displacement responses, it can be seen that the displacement-time histories were computed with high accuracy for the first four impact events (events TH5-1 through TH5-4).

However, the analyses pertaining to the following four impacts (TH5-5 through TH5-8) significantly overestimated the amount of slab damage that had occurred in these tests, resulting in high estimates of the peak and residual midpoint displacements. The support reaction responses were computed with similar levels of accuracy obtained for other RC and R/FRC slabs. The peak reaction forces were underestimated in all cases; however, reasonable agreement between the experimental and analytical impulse responses was obtained.

Although the analytical results presented in Figure 7.34 appear to be as equally stable as results obtained for other R/FRC slabs (e.g., slabs TH3, TH4, and TH8), the VecTor4 analyses of slab TH5 did not exhibit the same level of performance in terms of computational stability. In the analyses pertaining to slab TH5, the program required more iterative computations to achieve similar, or in most cases, less accurate levels of convergence and, as a result, required significantly more time to complete the analyses. It is not clearly evident if the increased fibre volume fraction (e.g.,  $V_f = 1.50$  % in this case) led to the reduction of the solution stability; however, further investigation regarding the FRC solution algorithm may be warranted.

#### *Slab TH8* ( $\rho_l = 0.592$ %; $V_f = 1.00$ %)

Figure 7.35 presents the analytical midpoint displacement-time, support reaction-time, and impulse-time histories for slab TH8. Comparisons between the experimental and the analytical responses show that the analyses of the initial impacts of the slab resulted in high accuracy estimates of the peak midpoint displacements. With the exception of the first impact test performed on the undamaged slab, the analytical results also provided good estimates of the residual displacements of the slab. As was the case with RC slab TH7, which also contained a longitudinal reinforcement ratio of 0.592 %, the periods of the responses were generally underestimated.



Figure 7.34 – Analytical Response-Time Histories; Slab TH5 ( $\rho_l = 0.420$  %;  $V_f = 1.50$  %)



Slab TH5 ( $\rho_l = 0.420$  %;  $V_f = 1.50$  %)



Figure 7.35 – Analytical Response-Time Histories; Slab TH8 ( $\rho_l = 0.592$  %;  $V_f = 1.00$  %)





With respect to the analytical results obtained for other R/FRC slabs, the support reaction responses were computed with reasonable accuracy. The general trends of the support reaction responses match that recorded from the experiments; however, peak reactions were greatly underestimated.

Analysis of the two final impact tests performed on the R/FRC slab (events TH8-7 and TH8-8) provided the least accurate estimates of the experimental behaviours. Although experimental midpoint displacement data were not obtained for event TH8-7, it can be seen from both the periods and the magnitudes of the support reaction and impulse responses (see Figure 7.35g) that the stiffness of the slab is underestimated in the analysis. Additionally, the development of numerical instabilities in the displacement-time history is apparent toward the end of the analysis. The results from the eighth and final impact test analysis significantly overestimate the amount of slab damage with overestimations of the response period, and the peak and residual midpoint displacements.

The midpoint displacement results for the R/FRC slabs have been summarized in Table 7.6. It can be seen that VecTor4 estimated the peak displacements of the slabs with high accuracy, obtaining a mean analytical-to-experimental displacement ratio of 1.07 with a coefficient of variation of 13 %. In general, analyses pertaining to the latter stages of the impact protocols provided less accurate results than those obtained from the analyses of the initial impact events. This was particularly evident in the analyses of R/FRC slab TH5.

The residual midpoint displacements were computed with significantly lower levels of accuracy than the peak displacements, and were routinely underestimated with few exceptions. When all of the R/FRC slab analyses are considered, the residual displacements were computed with a mean analytical-to-experimental ratio of 0.74 with a coefficient of variation of 83 %. However, as noted previously, given that the values of the residual displacements in many of the tests were relatively small, absolute error estimates may be more appropriate. The mean absolute residual error was calculated to be 2.0 mm with a standard deviation of 1.7 mm. Based on these values, it can be seen that the large inaccuracies in the computed residual displacements likely have limited significance for many of the impact tests.

	Peak Displacement (mm)			Residual Displacement (mm)			
Event	Test	VT4	VT4/Test	Test	VT4	VT4/Test	error
TH3-1	-	14.4	-	2.9	0.3	0.11	2.6
TH3-2	17.3	17.8	1.03	1.0	0.0	0.03	1.0
TH3-3	22.0	21.7	0.98	3.3	1.8	0.54	1.5
TH3-4	27.3	26.8	0.98	7.1	3.8	0.54	3.3
TH3-5	32.5	30.8	0.95	13.3	6.1	0.46	7.2
TH4-1	14.0	13.3	0.95	2.7	0.2	0.09	2.5
TH4-2	17.0	16.3	0.96	1.0	0.1	0.10	0.9
TH4-3	21.6	20.5	0.95	2.2	1.4	0.65	0.8
TH4-4	24.1	25.2	1.04	3.7	3.0	0.81	0.7
TH4-5	25.9	27.2	1.05	4.6	3.1	0.67	1.5
TH4-6	29.2	32.6	1.12	6.9	6.3	0.91	0.6
TH4-7	32.0	33.7	1.05	9.1	5.3	0.59	3.8
TH5-1	12.4	13.6	1.10	3.4	0.3	0.08	3.1
TH5-2	15.2	16.8	1.11	0.6	0.0	0.02	0.6
TH5-3	19.3	20.0	1.03	1.7	1.4	0.81	0.3
TH5-4	22.2	24.8	1.12	2.8	2.0	0.72	0.8
TH5-5	22.9	28.0	1.22	2.8	4.6	1.65	1.8
TH5-6	25.7	34.2	1.33	4.0	7.7	1.94	3.7
TH5-7	26.3	36.2	1.38	4.0	8.2	2.05	4.2
TH5-8	29.1	43	1.48	5.4	11.7	2.17	6.3
TH8-1	13.5	12.8	0.95	2.7	0.3	0.12	2.4
TH8-2	17.0	15.4	0.91	0.7	0.1	0.18	0.6
TH8-3	20.4	19.1	0.94	1.0	1.5	1.54	0.5
TH8-4	23.7	22.2	0.94	2.5	1.4	0.57	1.1
TH8-5	25.0	23.9	0.96	3.2	2.8	0.87	0.4
TH8-6	27.3	27.2	1.00	4.4	2.6	0.59	1.8
TH8-7	27.5	29.7	1.08	5.2	4.0	0.76	1.2
TH8-8	31.5	39.2	1.24	8.8	10.2	1.16	1.4
		Mean =	1.07		Mean =	0.74	2.0 mm
		C.O.V. =	13 %	C.O.V	$(\sigma_{res}^*) =$	83 %	(1.7 mm)

Table 7.6 - Summary of Midpoint Displacement Results for R/FRC Slabs

* standard deviation of absolute residual displacement error

A summary of the computed support reactions for the R/FRC slabs in presented in Table 7.7. As was found in the analyses of the RC slabs, the peak reactions were consistently underestimated in the analyses, with VecTor4 typically calculating peak reactions in the order of 50 % to 60 % of that measured experimentally.

	Peak Support Reaction (kN)			Peak	Peak Impulse [*] (kN-s)			
Event	Test	VT4	VT4/Test	Test	VT4	VT4/Test		
TH3-1	523	352	0.67	2.12	2.65	1.25		
TH3-2	684	427	0.62	2.69	3.12	1.16		
TH3-3	743	431	0.58	3.02	3.62	1.20		
TH3-4	637	346	0.54	3.54	3.84	1.08		
TH3-5	401	201	0.50	3.52	3.23	0.92		
TH4-1	591	368	0.62	2.00	2.65	1.33		
TH4-2	724	434	0.60	2.75	3.18	1.16		
TH4-3	703	434	0.62	3.52	3.62	1.03		
TH4-4	789	419	0.53	3.85	3.95	1.03		
TH4-5	701	372	0.53	3.34	3.91	1.17		
TH4-6	643	336	0.52	3.88	4.24	1.09		
TH4-7	591	290	0.49	4.15	3.95	0.95		
TH5-1	531	412	0.78	2.10	2.60	1.24		
TH5-2	725	425	0.59	2.70	3.19	1.18		
TH5-3	779	477	0.61	3.34	3.66	1.10		
TH5-4	756	449	0.59	3.68	3.97	1.08		
TH5-5	703	416	0.59	3.55	3.74	1.05		
TH5-6	777	370	0.48	4.05	4.10	1.01		
TH5-7	755	325	0.43	4.07	4.02	0.99		
TH5-8	766	315	0.41	4.47	4.40	0.98		
TH8-1	621	461	0.74	2.06	2.58	1.25		
TH8-2	721	507	0.70	3.02	3.13	1.04		
TH8-3	759	486	0.64	3.39	3.61	1.06		
TH8-4	772	459	0.59	3.74	4.00	1.07		
TH8-5	720	409	0.57	3.51	3.92	1.12		
TH8-6	767	348	0.45	3.92	3.95	1.01		
TH8-7	692	296	0.43	4.05	3.71	0.92		
TH8-8	684	240	0.35	4.25	3.70	0.87		
		Mean =	0.56		Mean =	1.08		
		C.O.V. =	17 %		C.O.V. =	10 %		

Table 7.7 – Summary of Support Reaction Results for R/FRC Slabs

* maximum value within first cycle of response

The impulse responses of the support reactions were computed with better accuracy. In comparing the analytical and the experimental peak impulses computed within the first cycle of the reaction force responses, a mean analytical-to-experimental ratio of 1.08 with a coefficient of variation of 10 % was obtained.

### Displaced Shapes and Failure Modes

The analytical displacement profiles of the R/FRC slabs are presented in Figure 7.36. The presented displaced shapes represent the slab deformations at the time of the peak computed midpoint displacement, and have been plotted against the peak experimental displacement profiles.



The displacement profiles from the analyses of the initial impacts matched those measured experimentally. However, as the VecTor4 analyses progressed through the sequences of impacts, agreement between the experimental and analytical displacements deteriorated. In contrast to the definitive punching failures which were computed in the analyses of the RC slabs, the analytical

displacement profiles for the R/FRC slabs showed that only limited degrees of punching were estimated to occur. For example, from the experimental displacement data presented in Figure 7.36a, it can be seen that punching under the impact load had occurred in slab TH3 after the fourth impact, and that the localized punch further progressed under the following fifth impact test (TH3-5). The analytical results do not indicate that localized punching ultimately controlled the behaviour of the slab. The computed displacement profile from event TH3-4 shows that some degree of localized punching was estimated to occur; however, the analysis of the following impact did not lead to further development of the already punched impact region. Similar behaviours were evident in the results of slabs TH4 and TH8.

Reasonable agreement was obtained between the experimental and analytical displacement profiles for slab TH5. Under the fifth impact, the development of localized punching was accurately estimated; however, analyses of additional impact greatly overestimated the overall displacement behaviour of the slab.

## Reinforcing Bar Strains

This subsection presents a brief comparison between the analytical and experimental reinforcing bar strains for the R/FRC slabs. Strain gauge S1, which is located in the midpoint region of the slab on the interior bottom-face mat of reinforcement, was considered. Results from selected impact events from the testing of TH3 and TH4 are provided.

The analytical bar strain-time responses for events TH3-2 through TH3-4 are presented in Figure 7.37. In comparing the computed responses with the experimental data, it is evident that only limited agreement was attained between the analytical and the measured bar strains. In the analysis of TH3-2, the peak bar strains were estimated reasonably well; however, the residual strains were greatly underestimated. Analysis of the following impact, event TH3-3, provided reasonable estimates of both the peak and the residual bar strains. Lastly, results from event TH3-4 significantly overestimated both the peak and the residual strains. Similar discrepancies on the computed reinforcing bar strain responses were found from the analytical results pertaining to slab TH4 (see Figure 7.38).



7.2.2.7 Summary of Analytical Slab Results

High levels of agreement between the computed and the measured responses were obtained for the RC and R/FRC slabs comprising the experimental program. The developed dynamic analysis subroutines performed well and were shown to reasonably estimate the behaviours of the slabs under sequential impact loading conditions. When the results from both the RC and the R/FRC slabs are considered, VecTor4 was found to achieve a mean analytical-to-experimental ratio of 1.06 with a coefficient of variation of 13 % for the computed peak midpoint displacements.

Although, in most cases, VecTor4 was found to remain stable throughout the performance of multiple consecutive impact analyses, the accuracy of the computed responses were found to deteriorate over the progression of the impact protocols. This reduction was particularly evident in the analyses of the later impact events pertaining to the R/FRC slabs. In Figure 7.39, the

analytical-to-experimental ratios for the peak midpoint displacements, peak support reaction forces, and the peak impulses are plotted with respect to the impact progression.



Figure 7.39 - Accuracy of Analytical Results over Impact Progression

From the figure, it is evident that the peak displacements and the peak reaction forces were computed with reduced accuracies in the later impact events when compared to the results from the initial events. However, the computed impulse seemed to provide reasonable estimates throughout the full progression of analyses.

As previously noted, some level of error accumulation is anticipated when performing consecutive finite element analyses which rely on the progressive development of material damage and progressive plastic material straining. However, this form of error accumulation is likely only partially responsible for the large levels of error encountered in the later impact analyses of the R/FRC slabs.

Presentations of the deformed shapes of the RC slabs revealed that VecTor4 accurately captured the localized punching behaviours ultimately governing the failures of the RC slabs, and provided reasonable estimates of the slabs' global deformation behaviours. However, the deformed shapes of the R/FRC slabs were not computed with the same high level of accuracy as that attained for the RC slabs. Additionally, the pronounced punching behaviours observed experimentally in the R/FRC slabs containing steel fibre volume fractions of 0.50 % and 1.00 % were not captured well. It is believed that in addition to the errors associated with the performance of consecutive impact analyses, these errors are most likely attributed to the hysteretic behaviour considered for the SFRC.

All analyses of the R/FRC slabs were performed considering a simple back-bone monotonic steel fibre-tension constitutive response. As such, permanent strains resulting from fibre slip and fibre pullout across the cracks have been neglected. Extension of the current fibre-tension model considered in VecTor4 such that strain offsets can be incorporated within the material response of SFRC subjected to load reversals may address the deficiencies found in the analyses of the R/FRC slabs.

## 7.2.3 Saatci Beams

Selected beam impact tests from a recent experimental program performed by Saatci and Vecchio (2009a) at the University of Toronto have been analyzed using VecTor4. As noted previously, this beam testing program was used as the basis in designing the slab impact experiments and, although similar testing methodologies were used in both programs, key differences regarding the specimens and the loading conditions make the Saatci beams a useful data set for verifying the dynamic analysis capabilities of VecTor4.

The testing program consisted of eight simply-supported RC beams subjected to falling weight impacts. The beams were impacted at their midspans, and were supported using a similar roller configuration to that used in the slab experimental program presented in this thesis (refer to Chapter 3). The beams possessed uniform geometries and longitudinal reinforcement schemes (see Figure 7.40); however, the shear reinforcement ratio was varied from zero to 0.30 %. The beams were designed and cast as four series (series SS0, SS1, SS2, and SS3) of twinned specimens. The twin specimens were identical in all aspects, but were subjected to different impact loading protocols: protocol 'a', or protocol 'b'.

The beams were symmetrically reinforced with two 30M longitudinal bars on each face of the cross section, and shear reinforcement was provided in the form of closed-hoop D6 stirrups. The beams were cast using ready-mix concretes with 50 MPa nominal compressive strengths and a maximum aggregate size of 10 mm. The material properties of the steel reinforcing bars obtained from coupon testing are presented in Table 7.8, and the material properties of the concretes evaluated at the time of beam testing are presented in Table 7.9.


Figure 7.40 – Geometric and Reinforcement Details for Saatci Beams (adapted from Saatci and Vecchio, 2009a) (dimensions in millimetres)

Table 7.8 –	Reinforci	ing Bar	Material	<b>Properties</b>	for	Saatci	Beams

Rebar	$A_s$ (mm ² )	d _b (mm)	$f_y$ (MPa)	$f_u$ (MPa)	Es (MPa)	$\mathcal{E}_{sh}$ (x 10 ⁻³ )	$\mathcal{E}_u$ (x10 ⁻³ )
M30	700	29.9	464	630	195,000	12.5	165
D6	38.7	7.0	605	652	190,250	3.2	20

Specimen	<i>f_c</i> , (MPa)	$\mathcal{E}_c$ ' (x 10 ⁻³ )	E _{cs} (MPa)	$f_r^*$ (MPa)
SS0a, SS0b	50.1	2.32	32,000	5.90
SS1a, SS1b	44.7	2.36	27,000	5.60
SS2a, SS2b	47.0	2.42	28,000	6.20
SS3a, SS3b	46.7	2.51	27,000	5.80

Table 7.9 - Concrete Material Properties for Saatci Beams

note: maximum nominal aggregate size = 10 mm for all specimens *from modulus of rupture prisms

The general focus of the experimental program was to investigate the influence of shear mechanisms on the impact responses of RC beams. SS0 and SS1 series beams were designed such that they were shear-critical under conventional monotonic loading conditions, and beam series SS2 and SS3 were designed such that they were controlled by flexural failure mechanisms.

Similar to the results from the slab impact testing program, Saatci and Vecchio found that all of the RC beams, including those which were flexure-critical, were controlled by shear failure mechanisms when tested under high-mass low-velocity impacts.

Two impact loading protocols were considered in the testing program: a) an initial impact of 211 kg followed by two additional impacts using a drop-weight of 600 kg, or b) two impacts of 600 kg followed by a third impact of 211 kg. A constant nominal impact velocity of 8.0 m/s was considered for all of the tests.

VecTor4 analyses of the first two impact events were performed for four of the beams from the experimental program: SS1a, SS2a, SS1b, and SS3b. Taking advantage of symmetry, quarterbeam finite element models were considered in the analyses. The finite element mesh used for the analyses of the Saatci beams is presented in Figure 7.41. Eleven shell elements were used to represent the RC beams, and an additional twelfth shell element, which is not shown in the figure, was used to model the rigid drop-weight. The impacting drop-weight was connected to the midspan nodes of the beam using three 'compression-only' linear elastic truss elements. The truss elements were assigned high stiffness values ( $k_{truss} = 5 \times 10^6 \text{ kN/mm}$ ) such that hard impact conditions were considered in the analyses.



Figure 7.41 – Finite Element Mesh Used for Saacti Beams (dimensions in millimetres)

Similar to the modelling approach considered in the analysis of the impact slabs, out-of-plane shear strength enhancement was provided in the region surrounding the location of the support reaction, and in the mass impact region. Out-of-plane shear strains were reduced by 50 % within the enhanced regions.

The shell elements comprising the beam were subdivided into 25 concrete layers, and an additional four layers were used to represent the longitudinal reinforcement components and the horizontal reinforcement components provided by the closed stirrups. The top and bottom concrete cover of the beam was taken as 35 mm, and out-of-plane shear reinforcement was treated as a smeared property of the non-cover concrete layers.

The beams were analyzed using default behavioural models with one exception: the reinforcement hysteresis was modelled using an elastic-plastic model with curvilinear hardening, for reasons discussed previously (refer to Section 7.2.2.1). The direct tensile strengths of the beams were approximated as  $0.33\sqrt{f_c}$ , the densities of the beams were taken as 2,400 kg/m³, and Popovics HSC model was selected as the compression base curve for the 50 MPa nominal strength concretes.

Similar to those performed for the slab analyses, parametric investigations were used to select appropriate dynamic analysis parameters for the beams. Supplemental damping was provided in the form of stiffness damping, and was applied uniformly for all beams and impact events. A stiffness proportional damping coefficient value of  $0.52 \times 10^{-4}$  s was determined to effectively stabilize the beam analyses. The time step used in the analyses was selected as 0.00001 s, and Newmark's average acceleration method was used to perform the numerical time integration.

The results from the initial impacts of the Saatci beams are presented in Figure 7.42. It can be seen that although VecTor4 typically underestimated the peak midspan displacements, the peak values were estimated reasonably well. The residual displacements were also typically underestimated; however, the computed residual displacements from the analyses of the 600 kg impact events (i.e., SS1b-1, and SS3b-1) were in good agreement with experimental values.

Comparing the computed support reaction-time histories to those measured experimentally, it can be seen that the peak reaction forces were overestimated in the 211 kg impacts, but were computed with good accuracy in the 600 kg impact tests. The computed reaction-time responses contained appreciable high frequency contributions which were not present in the measured experimental responses. Spurious modes may be responsible for erroneous contributions in the reaction time-histories, and could be potentially suppressed with additional viscous damping.



Figure 7.42 – Analytical Response-Time Histories for Saatci Beams; First Impacts

Lastly, examination of the calculated support reaction impulses shows that although the frequency contents of the computed reaction responses differed significantly from the experimental results, both the magnitudes and the general shapes of the reaction impulses were computed with good agreement when compared with experimental measurements.

The results from the VecTor4 analyses of the second impact events of the four Saatci beams are presented in Figure 7.43. Overall, it can be seen that the behaviours of the already-damaged RC beams were estimated with similar levels of accuracy as that of the undamaged beams. Accurate estimates of the peak midspan displacements were achieved for tests SS1a-2, SS2a-2, and SS3b-2; however, the midspan displacement amplitude overestimated the failure behaviour of Beam SS1b-2 (see Figure 7.43c).

The values of the peak support reactions were also captured reasonably well in the analyses of the second impact tests, but still possessed high frequency response contributions which were not measured experimentally. These high frequency contributions may be attributed to the simplified approach used to model the support conditions used in the experimental program. In comparing the analytical impulse-time histories with those computed from experimental data it can be seen that the impulse responses of the 'a'-series beams were estimated with high accuracy in terms of both their magnitudes and their response frequencies. The analytical impulses of the 'b'-series beams however, provided tended to overestimate the impulse magnitudes and underestimate the reaction response periods.

It can be seen that in general, the behaviours of the Saatci and Vecchio beams were estimated with reasonable levels of accuracy using VecTor4. With the exception of impact event SS1b-2 where extensive damage had occurred as a result of a shear failure, estimates of the peak midspan displacements closely matched those measured experimentally. The support reaction-time histories were computed with less accuracy, but the peak reaction values were computed well in most cases and the analytical impulses showed good agreement with the experimental behaviours.



Figure 7.43 – Analytical Response-Time Histories for Saatci Beams; Second Impacts

### 7.3 Chapter Summary and Conclusions

In this chapter the methodologies used to implement dynamic analysis capabilities within VecTor4 were presented and discussed. An overview of the evaluation of dynamic system properties such as the damping matrix, mass matrix, and the dynamic load vector was provided, and the numerical solution method used to perform the required time-stepping analysis procedure was summarized. Advanced material models used to accommodate material strain rate effects were added to the program, and their influence on the computed response of an RC slab subjected to impact loading conditions was investigated.

The newly implemented dynamic analysis features of VecTor4 were assessed using a series of linear elastic verification studies, as well as test data from experimental programs. The results from the linear elastic analyses showed that the program was effective in modelling different types of dynamic loading conditions such as ground motions, user-defined impulse loads, or prescribed mass motion parameters (e.g., initial velocity, constant acceleration). It was also found that the implemented element mass lumping procedure served as an effective method in distributing mass throughout the complex high-order shell elements employed in VecTor4. However, the results from a series of linear elastic thick-plate analyses suggested that solution accuracy might be further improved by incorporating rotational inertia contributions within the assembled mass matrices.

Data from two experimental testing programs were used to verify the nonlinear dynamic analysis capabilities of VecTor4: a series of shear-critical RC beams subjected to drop-weight impacts, and the RC and R/FRC impact slab tests forming the experimental research component of this thesis. In general, VecTor4 provided reliable estimates of the peak displacements of the RC and R/FRC members subjected to the high-mass low-velocity impact loading conditions. The accuracy of the displacement estimates tended to deteriorate over the course of analyzing the progressive impact protocols used in the experimental programs; however, the accuracies only significantly decreased in the analyses of the R/FRC slabs which were subjected to several impacts prior to failure. When the peak displacement results from all 36 impact analyses performed are considered, an analytical-to-experimental value of 1.05 with a coefficient of variation of 13 % is attained.

The support reaction responses were computed with a lower degree of accuracy than that of the displacements. In the case of the slab analyses the peak reaction forces were consistently underestimated, but the overall shapes of the responses agreed reasonably well with the experimental data. The analytical support reaction-time histories of the RC beams were dominated by high frequency contributions which were not measured experimentally and, as such, the overall reaction responses appeared to be significantly different the experimental responses. However, the computed impulse-time histories of both the beams and the slabs were computed with high accuracies and, in most cases, the peak impulse value was in good agreement with that measured during the tests.

In both the analyses of the beams and the slabs, the damping behaviours of the free-vibration responses were typically underestimated by VecTor4. The analyses were performed using minimal levels of supplemental stiffness proportional damping for the purpose of stabilizing the solution algorithm and, as a result, the computed damping behaviours were primarily controlled by the nonlinear hysteretic responses of the materials comprising the structures. The amount of additional viscous damping supplied was determined through the performance of limited parametric investigations. Future studies focused on developing more rational approaches of evaluating appropriate supplemental damping may be warranted.

Lastly, the use of Newmark's average acceleration method was considered in all of the analyses comprising the dynamic verification study. Although this method was selected to be the most suitable approach amongst those which were implemented within VecTor4 (e.g., Newmark's average acceleration method, Newmark's linear acceleration method, Wilson's theta method), it required the use of extremely small time steps making the procedure somewhat prohibitive. Alternative methods such as the HHT- $\alpha$  solution method (Hilber et al., 1977) which can be altered to introduce numerical damping could potentially be used to improve solution accuracy and maintain solution stability without decreasing the size of the time-step (Paultre, 2011).

## CHAPTER 8: CONCLUSIONS AND RECOMMENDATIONS

The primary goal of this thesis was to further develop the analytical software program VecTor4: a nonlinear finite element program dedicated to the analysis of reinforced concrete slabs and shell structures. In its previous condition, VecTor4 was of somewhat limited use due to its inability to accurately capture out-of-plane shear failures and because analysis capabilities were restricted to force-controlled monotonic loading conditions. In the process of redeveloping and improving the performance of the program, this thesis focused on addressing the following analytical objectives:

- 1. Identifying and correcting deficiencies pertaining to the existing version of VecTor4.
- 2. Implementing and verifying the performance of new general features and analysis options which extend the range of structure types, loading conditions, and behavioural mechanisms that can be considered in the VecTor4 analyses.
- 3. Developing and verifying the performance of cyclic and dynamic analysis capabilities within VecTor4.

To verify the performance of the implemented dynamic analysis capabilities (item 3 in the above-stated objectives), an experimental testing program focused on the behaviour of reinforced concrete (RC) and fibre-reinforced concrete (R/FRC) slabs under impact loading conditions was performed. In addition to appraising the analytical procedure, the experimental program was carried out to provide a well-documented data set which could be used by others to assess the performance of currently existing and future developed analytical procedures, and to provide data related to an emerging area of research where currently only limited exist: the global response of RC and R/FRC slabs under impact.

### 8.1 Conclusions

This section presents the main conclusions from the research program. For clarity, findings from the experimental and the analytical investigations have been presented separately.

With respect to the experimental program, the following conclusions can be drawn:

- i. Although all of the slabs forming the experimental program were designed to be governed by flexural failure modes under static loading conditions, nearly all of the slabs were controlled by punching shear failures under impact.
- ii. The initial phases of the impacts were almost entirely resisted by the inertias of the slabs.Peak support reactions were not in-phase with peak impact forces.
- iii. Increasing the longitudinal reinforcement ratio had limited influence on the impact responses of the slabs. For example, Slabs TH2 ( $\rho_l = 0.420$  %) and TH7 ( $\rho_l = 0.592$  %) exhibited similar performance characteristics in terms of strength, stiffness, post-peak vibration, and local damage.
- iv. The R/FRC slabs exhibited superior performance characteristics under impact loading conditions when compared to conventional RC slabs. The use of steel fibre-reinforced concretes led to:
  - reduced crack spacings and reduced crack widths;
  - mitigation of local damage (e.g., mass penetration, concrete scabbing);
  - increased stiffnesses and increased capacities.
- v. The improved performance characteristics of the R/FRC slabs under impact loading correlated with the steel fibre volume fraction provided.
- vi. R/FRC slab TH5, which contained the largest fibre volume fraction considered in this study ( $V_f = 1.50$  %), was the only slab which was effective in preventing punching from occurring under the prescribed impact loading protocol. Local damage was limited to minimal mass penetration, and the slab underwent large displacements and experienced extensive longitudinal steel yielding without failing.

To assess the quality of the digital data obtained from the experimental testing program, a signal response investigation was undertaken. The following general observations were found from the investigation:

 data pertaining to slab displacements, load cell reaction forces, and reinforcing strains were adequately collected;

- strain measurements pertaining to the support reaction assemblies were collected sufficiently well; however, improved resolutions may have been obtained from increased sampling rates;
- accelerometers used to measure the impact mass accelerations often saturated, rendering the data unusable. Accelerations of the slab were captured more reliably.

With respect to the development and performance of program VecTor4, the analytical component of this study, the following general conclusions can be drawn:

- The nonlinear finite element procedure developed in accordance with the constitutive relations of the Disturbed Stress Field Model served as a viable approach for the analysis of RC shells and slabs under general loading conditions.
- High accuracy response estimates pertaining to RC beams, slabs, shells, thin-plates, and R/FRC panels subjected to monotonic loading conditions were achieved using basic finite element modelling techniques, default (basic) material models, and default analysis parameters.
- iii. Shell finite element sizing had little impact on member strength estimates; however, ductility estimates exhibited some level of mesh dependency. The number of layers used to represent the cross sections of the RC shell elements was found to marginally influence the computed results.
- iv. The modification of VecTor4's thick-shell formulation provided improved out-of-plane capacity estimates for shear-critical members. The modification method developed ensures that the shell element sectional forces equilibrate global system forces, regardless of out-of-plane shear strain assumptions.
- v. The Simplified Diverse Embedment Model was found to be a viable modelling approach for the analysis of R/FRC elements constructed using end-hooked steel fibres.

Analytical conclusions specific to the implementation of dynamic analysis capabilities within VecTor4 are as follows:

vi. The analysis framework developed was capable of providing reliable assessments for RC and R/FRC elements subjected to high-mass low-velocity impact loading conditions.

Using basic modelling techniques and simple finite element meshes, high accuracy estimates were obtained for the midpoint displacement-time histories and the displaced shapes of the slabs comprising the experimental program. Similar levels of accuracy were obtained for analyses pertaining to RC beams under impact.

- vii. The punching shear failure modes which governed the behaviours of the slabs comprising the experimental program were accurately captured in the analyses.
- viii. Peak support reaction forces were estimated with less accuracy; however, the general shapes of the reaction force-time histories and the computed support reaction impulses computed for the slabs were in good agreement with those measured experimentally.
  - ix. The computed post-peak vibrations of the slabs provided reasonable estimates of the experimental response frequencies. However, discrepancies existed between the analytical and the experimental post-peak damping characteristics, with computed analytical results typically underestimating apparent damping.
  - x. The analysis method proved to be capable of performing sequential impact analyses on damaged specimens. In general, analyses of the RC slab and beam specimens remained stable throughout the multiple event impact loading protocols. Instabilities were observed in the later-event analyses of the R/FRC slabs.
  - xi. Although it required the use of relatively small time steps, Newmark's average acceleration method was shown to be a suitable time integration method for multi-degreeof-freedom nonlinear dynamic finite element analyses. The influence of time step size primarily affected solution accuracy, and exhibited little effect on solution stability.
- xii. The implemented HRZ mass lumping scheme was determined to be a viable approach for distributing mass amongst complex high-order finite elements.
- xiii. Lastly, minimal stiffness proportional damping assignments effectively stabilized the analyses. Constant stiffness proportional damping assignments used for all specimens and impact events forming the beam and slab testing programs should be viewed as a practical approach for assigning supplemental damping.

### 8.2 Recommendations for Future Work

While the work undertaken in this thesis was successful in accomplishing the outlined objectives, there were a number of testing procedures which would likely benefit from alteration in future studies. Similarly, a number of limitations and deficiencies were identified over the course of the analytical development and assessment. Recommendations which are believed to represent improved testing procedures and further analytical development of VecTor4 are summarized below:

With respect to the experimental program:

- i. The use of accelerometers to characterize the impact force presented several challenges as they often saturated, rendering the data unusable, and they required extensive post-test filtering. More reliable impact force data may be obtained using a high capacity shockrated load cell.
- The performance of complimentary monotonic/static slab tests would be useful in defining whether computed analytical errors were a function of the newly implemented dynamic analysis subroutines or were associated with the general modelling approach. Additionally, data from static testing could also be used to assess the functionality of the support conditions used in the experimental program.
- RC slabs constructed with conventional out-of-plane shear reinforcement tested under impact would provide additional data to evaluate the relative effectiveness of the R/FRC slabs.
- iv. The potentiometers used to measure slab displacement data were often damaged as a result of the initial shocks developed under the applied impact loads. More robust sensors with appropriate shock capacities would be recommended for future studies.
- v. An insufficient number of accelerometers were used to quantify the inertial response of the slabs. Future investigations involving slabs under impact, and especially those affected by shear-related mechanisms leading to non-uniform deformations, should consider alternative methods to measure specimen accelerations.

With respect to the development of the software program VecTor4:

- i. The parabolic out-of-plane shear strain assumption was found to capture the out-of-plane shear response of RC members with reasonable levels of accuracy. However, the response of shear-critical members may be further improved with a more rigorous treatment of the out-of-plane sectional shear behaviour.
- ii. Although the methodology was intended to be approximate, the use of the out-of-plane shear strength modification (SSM) factors to accommodate disturbed regions may benefit from further study. Ideally, disturbed regions should be considered in all analysis types (e.g., monotonic, cyclic, and dynamic) and should be assigned automatically within the program.
- iii. Post-peak compression response modelling within VecTor4 requires further investigation. The current methodology tends to underestimate the ductilities of flexure controlled members and propagates artificial sectional shear failures.
- iv. An appropriate R/FRC hysteresis model should be implemented within the program.
- v. Additional monotonic and cyclic loading verification considering R/FRC structures should be performed to identify deficiencies in current FRC modelling procedures.
- vi. The strain rate effect models forming the default analysis options in VecTor4 were primarily selected on the basis of investigations performed by other researchers employing similar methodologies. A more critical assessment of appropriate material strain rate enhancement models should be performed using VecTor4.
- vii. Results from the linear elastic verification study suggested that improved accuracies may be obtained for thick-shell elements if rotational inertia effects are incorporated within the element mass matrix. Alternative mass lumping procedures which include rotational inertia should be implemented.
- viii. Alternative concrete hysteresis models which consider damage accumulation from low amplitude cycling should be investigated. Such models may better estimate the damping characteristics of RC elements under impact.
  - ix. Lastly, alternative time integration procedures which are capable of incorporating numerical damping and provide improved solution accuracies when compared with Newmark's average acceleration method should be implemented within the program.

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# **APPENDIX A**: INSTRUMENTATION PLANS

Appendix A presents additional information and plans pertaining to the instrumentation provided in the experimental program.

### A1: Accelerometer

Section A1 of the appendix presents the accelerometer instrumentation plans used in the slab impact testing program. A typical instrumentation plan was essentially used for all eight test slabs with only minor deviations.

## A2: Potentiometer

In section A2 of the appendix, potentiometer instrumentation plans have been provided. For several reasons as noted in Chapter 4, the placement of the potentiometers varied over the course of the experimental program. Additionally, potentiometer data which were discarded due to damage, or for other reasons, have been tabulated and summarized for each impact event.

## A3: Strain Gauges

Section A3 presents the instrumentation plans for the strain gauges installed on the reinforcing bars within the test slabs. Typical plans have been presented. Note that not all gauges were used in all tests.



## **A2:** Potentiometers



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(A2 - continued)

	Discarded Sensor ¹		Discarded Sensor ¹		W. Support	لى ا
Event	(potentiometer #)	Event	(potentiometer #)		Location	Q P11
TH1-1	1, 2, 6, 7, 13, 14	TH5-4			_	
TH1-2	2	TH5-5				- 022 0 00
TH1-3		TH5-6				(c) = (c) = (c)
TH2-1	1, 7, 13	TH5-7				
TH2-2	6, 14, 15, 19	TH5-8				– O P13
TH2-3	5	TH5-9			D2 D4 D5	$\mathbf{p}_{\mathbf{f}} = \mathbf{p}_{\mathbf{f}}^{2}$
TH3-1	filter settings ²	TH5-10		P22 O	сф t t t t t t t t t t t t t t t t t t t	$- \bigcirc - \bigcirc$
TH3-2		TH6-1		(lateral)	$(9) \underline{a} 189$	
TH3-3	7, 14	TH6-2	2, 8, 19		)	0 P14
TH3-4		TH7-1			Ċ	(c) (a) 189
TH3-5		TH7-2	1, 5, 6, 7		5	
TH4-1		TH7-3	6, 7, 8, 9, 14, 15, 17, 18, 19			378
TH4-2	2, 6, 14	TH8-1			-	
TH4-3		TH8-2	1, 6		+ <u></u>	
TH4-4		TH8-3	1, 4, 6, 7		2	
TH4-5		TH8-4				O P23 (later
TH4-6		TH8-5	1, 6, 7			
TH4-7		TH8-6		* P2	0 data collected usin	g QuantumX DAS
TH5-1		TH8-7	1, 2			1
TH5-2	1, 13	TH8-8	5, 6, 7, 13, 14, 18, 19			
TH5-3	13					
					Evanta TU7 J	and TUD 2. Cla





#### O P24 (lateral) $30 P_{P26}^{\downarrow}$ 175 [<del>``</del> 0 F O O Q P25 $^{-}$ $^{-}$ $^{-}$ 340 OP19 I ≊¢ O P21 (lateral) O P23 (lateral) I ЪO م ص ф Р12 O P13 0 P14 Ó P15 ⊕ P16 Ы -Nog $(5) @ 189^{1}$ 378 9°С O P17 Ó P20 ъq (9) @ 189 Support Location 40 සු

Events TH2-2 and TH2-3; Slabs TH3 – TH8 (all dimensions in millimetres)



## A3: Strain Gauges

## (A3 – continued)



Slab TH6 (dimensions in millimetres)

# **APPENDIX B**: COMPANION TEST RESULTS

Appendix B provides additional information regarding the material test results forming the companion studies.

### **B1:** Cylinder Compression Tests

Within section B1 of this appendix, the results from the concrete cylinder tests are presented. The tests were performed to characterize the compressive behaviour of the concrete materials. A set of ten cylinders were used to represent each of the eight slab specimens tested in the experimental program. The compressive stress,  $f_c$ , versus compressive strain,  $\varepsilon_c$ , behaviour is presented for three cylinders from each test slab. Key characteristic properties pertaining to each of the parent slabs have been tabulated.

### **B2:** Prism Bending Tests

In section B2 of this appendix, the results from the concrete bending prism tests are presented. The tests were performed to evaluate the flexural tensile behaviour of the concrete used in the experimental program. In the case of SFRCs, the post-peak bending behaviours were used to assess the influence of the steel fibres. Key characteristic properties pertaining to each of the parent slabs have been tabulated.

### **B3:** Uniaxial Tension Tests

Section B3 of this appendix presents the results from the uniaxial tension tests performed on the SFRC 'dog-bone' specimens. The 'dog-bone' tests were used investigate the direct tensile behaviour of the SFRCs comprising the R/FRC slabs constructed in the experimental program. Behaviours were investigated by way of pre-peak tensile stress versus strain behaviours, and post-peak tensile stress versus crack width opening behaviours. Characteristics pertaining to the parent R/FRC slabs have been tabulated.

## **B1:** Cylinder Compression Tests

## **B1.1 TH1 Cylinders** ( $V_f = 0$ )



Concrete Batch	<i>fc</i> ' 28 days (MPa)	$f_c$ ' test ¹ test age: 284 days (MPa)	<b>ɛ</b> ² _c (x 10 ⁻³ mm/mm)	<i>E_{cs}</i> (MPa)
B1	31.0	38.2	-	-
B2	60.2	76.7	2.388	43,000
B3	28.5	38.8	2.216	25,600
B4	50.4	71.7	2.592	38,400
B5	51.5	74.0	-	-
Mean	44.3	59.9	2.399	35,700
Std. Deviation	13.86	19.59	0.188	9,020
C.V. (%)	31.27	32.72	7.85	25.27
Mean ²	54.0	74.1	2.490	40,700
<b>Std. Deviation</b> ²	5.37	2.49	0.144	3,250
<b>C.V. (%)</b> ²	9.94	3.37	5.79	7.99

¹cylinders tested at time of slab impact testing ² based on test results from cylinders B2, B4, and B5 only

## B1.2 TH2 Cylinders ( $V_f = 0$ )



Concrete Batch	<i>fc</i> ' 28 days (MPa)	$f_c$ ' test ¹ test age: 318 days (MPa)	<b>ɛ²</b> _c (x 10 ⁻³ mm/mm)	E _{cs} (MPa)
B1	56.9	67.7	2.519	36,400
B2	58.5	70.8	-	-
B3	58.7	68.7	2.479	37,600
B4	57.6	69.8	2.665	36,400
В5	58.3	70.0	-	-
Mean	58.0	69.4	2.554	36,800
Std. Deviation	0.74	1.22	0.098	690
C.V. (%)	1.28	1.75	3.83	1.88

¹cylinders tested at time of slab impact testing
B1.3 TH3 Cylinders (*V_f* = 0.50 %)



Concrete Batch	<i>fc</i> ' 28 days (MPa)	$f_c$ ' test ¹ test age: 304 days (MPa)	<b>ɛ</b> ² _c (x 10 ⁻³ mm/mm)	E _{cs} (MPa)
B1	38.0	39.2	-	-
B2	38.2	48.4	2.743	27,400
B3	41.0	50.7	2.957	27,000
B4	39.2	49.6	2.705	32,000
В5	43.0	52.1	-	-
Mean	39.9	48.0	2.802	28,800
Std. Deviation	2.11	5.09	0.136	2,780
C.V. (%)	5.29	10.60	4.85	9.65

# B1.4 TH4 Cylinders (*V_f* = 1.00 %)



Concrete Batch	<i>fc</i> ' 28 days (MPa)	$f_c$ ' test ¹ test age: 340 days (MPa)	<b>ɛ²</b> _c (x 10 ⁻³ mm/mm)	<i>E_{cs}</i> (MPa)
B1	40.3	46.2	3.360	24,700
B2	40.9	49.7	-	-
B3	39.0	46.7	3.357	25,200
B4	44.3	50.0	3.170	26,900
В5	42.0	50.5	-	-
Mean	41.3	48.6	3.296	25,600
Std. Deviation	2.00	1.99	0.109	1,150
C.V. (%)	4.83	4.09	3.30	4.49

## B1.5 TH5 Cylinders (V_f = 1.50 %)



Concrete Batch	<i>fc</i> ' 28 days (MPa)	$f_c$ ' test ¹ test age: 340 days (MPa)	<b>ɛ</b> ²c (x 10 ⁻³ mm/mm)	E _{cs} (MPa)
B1	59.0	62.0	6.756	25,500
B2	46.0	49.0	4.193	21,300
B3	44.9	49.9	-	-
B4	52.9	57.6	4.183	25,200
В5	41.0	46.3	-	-
Mean	48.8	53.0	5.044	24,000
Std. Deviation	6.40	5.86	1.211	1,910
C.V. (%)	13.12	11.07	24.00	7.96
Mean ²	46.2	50.7	4.188	23,250
<b>Std. Deviation</b> ²	4.29	4.20	0.005	1,950
<b>C.V. (%)</b> ²	9.29	8.27	0.12	8.39

¹cylinders tested at time of slab impact testing ² based on test results from cylinders B2, B3, B4, and B5 only

# B1.6 TH6 Cylinders ( $V_f = 0$ )



Concrete Batch	<i>fc</i> ' 28 days (MPa)	$f_c$ ' test ¹ test age: 284 days (MPa)	<b>ɛ²</b> c (x 10 ⁻³ mm/mm)	<i>E</i> _{cs} (MPa)
B1	52.5	55.6	2.574	33,300
B2	58.5	58.7	2.736	32,200
B3	54.7	61.1	-	-
B4	56.9	60.4	-	-
В5	55.0	59.1	2.701	30400
Mean	55.5	59.0	2.670	32,000
Std. Deviation	2.28	2.12	0.085	1,460
C.V. (%)	4.11	3.59	3.19	4.56

# B1.7 TH7 Cylinders ( $V_f = 0$ )



Concrete Batch	<i>fc</i> ' 28 days (MPa)	$f_c$ ' test ¹ test age: 319 days (MPa)	<b>ɛ²</b> c (x 10 ⁻³ mm/mm)	Ecs (MPa)
B1	52.5	61.3	-	-
B2	55.0	63.0	2.837	34,000
B3	56.8	53.9	-	-
B4	54.0	61.5	2.503	31,800
В5	56.1	61.7	2.566	38,000
Mean	54.9	60.3	2.635	34,600
Std. Deviation	1.70	3.63	0.177	3,140
C.V. (%)	3.11	6.02	6.73	9.08

# B1.8 TH8 Cylinders (V_f = 1.00 %)



Concrete Batch	<i>fc</i> ' 28 days (MPa)	$f_c$ ' test ¹ test age: 353 days (MPa)	<b>ɛ</b> ² _c (x 10 ⁻³ mm/mm)	E _{cs} (MPa)
B1	45.7	48.9	3.120	26,700
B2	42.1	44.6	3.540	22,800
B3	40.8	43.0	3.440	22,600
B4	41.0	44.7	-	-
В5	44.6	44.4	-	-
Mean	42.8	45.1	3.367	24,000
Std. Deviation	2.20	2.24	0.219	2,310
C.V. (%)	5.14	4.95	6.52	9.63

#### **B2: Prism Bending Tests**

## **B2.1 TH1 Prisms** ( $V_f = 0$ )



Concrete Batch	<b>P</b> _{cr} ¹ (kN)	<b>P</b> _u (kN)	$P_u/P_{cr}$	$\delta_{cr}^{l}$ (mm)	$\delta_u^2$ (mm)	$\delta_u/\delta_{cr}$
B2	67.7	67.7	1.00	0.092	0.092	1.00
B4	62.0	62.0	1.00	0.079	0.079	1.00
Mean	64.9	64.9	-	0.086	0.086	-
Std. Deviation	4.03	4.03	-	0.009	0.009	-
C.V. (%)	6.21	6.21	-	10.47	10.47	-

# **B2.2 TH2 Prisms** ( $V_f = 0$ )



Concrete Batch	<b>P</b> _{cr} ¹ (kN)	<b>P</b> _u (kN)	$P_u/P_{cr}$	${\delta_{cr}}^{l}$ (mm)	$\delta_u^2$ (mm)	$\delta_u/\delta_{cr}$
B2	67.6	67.6	1.00	0.086	0.086	1.00
B3	65.5	65.5	1.00	0.084	0.084	1.00
B4	64.5	64.5	1.00	-	-	-
Mean	65.9	65.9	-	0.085	0.085	-
Std. Deviation	1.58	1.58	-	0.001	0.001	-
C.V. (%)	2.4	2.4	-	1.18	1.18	-

B2.3 TH3 Prisms (V_f = 0.50 %)



Concrete Batch	<b>P</b> _{cr} ¹ (kN)	<b>P</b> _u (kN)	$P_u/P_{cr}$	${\delta_{cr}}^{l}$ (mm)	$\delta_u^2$ (mm)	$\delta_u/\delta_{cr}$
B1	53.3	53.9	1.01	0.103	1.040	10.10
B3	54.1	54.1	1.00	0.099	0.099	1.00
В5	52.2	52.2	1.00	0.095	0.095	1.00
Mean	53.2	53.4	-	0.099	0.411	-
Std. Deviation	0.95	1.04	-	0.004	0.544	-
C.V. (%)	1.79	1.95	-	4.04	132.36	-

 1  development of first crack  2  displacement corresponding to peak load,  $P_{u}$ 

## B2.4 TH4 Prisms (V_f = 1.00 %)



Concrete Batch	<b>P</b> _{cr} ¹ (kN)	<b>P</b> _u (kN)	$P_u/P_{cr}$	${\delta_{cr}}^{l}$ (mm)	$\delta_u^2$ (mm)	$\delta_u/\delta_{cr}$
B2	47.1	61.8	1.31	0.089	1.299	14.60
B4	56.0	64.1	1.14	0.090	0.596	6.62
B5	50.8	59.6	1.17	0.091	0.359	3.95
Mean	51.3	61.8	-	0.090	0.751	-
Std. Deviation	4.47	2.25	-	0.001	0.489	-
C.V. (%)	8.71	3.64	-	1.11	65.11	-

 1  development of first crack  2  displacement corresponding to peak load,  $P_{u}$ 

# B2.5 TH5 Prisms (V_f = 1.50 %)



Concrete Batch	<b>P</b> _{cr} ¹ (kN)	<b>P</b> _u (kN)	$P_u/P_{cr}$	${\delta_{cr}}^{l}$ (mm)	$\delta_u^2$ (mm)	$\delta_u/\delta_{cr}$
B2	49.0	65.7	1.34	0.114	1.602	14.05
B4	56.8	69.9	1.23	0.109	1.108	10.17
Mean	52.9	67.8	-	0.112	1.355	-
Std. Deviation	5.52	2.97	-	0.004	0.349	-
C.V. (%)	10.43	4.38	-	3.57	25.76	-

# **B2.6 TH6 Prisms** ( $V_f = 0$ )



Concrete Batch	<b>P</b> _{cr} ¹ (kN)	<b>P</b> _u (kN)	$P_u/P_{cr}$	$\delta_{cr}^{l}$ (mm)	$\delta_u^2$ (mm)	$\delta_u/\delta_{cr}$
B1	56.2	56.2	1.00	0.069	0.069	1.00
B3	64.8	64.8	1.00	-	-	-
В5	62.8	62.8	1.00	0.083	0.083	1.00
Mean	61.3	61.3	-	0.076	0.076	-
Std. Deviation	4.50	4.50	-	0.010	0.010	-
C.V. (%)	7.34	7.34	-	13.16	13.16	-

# **B2.7 TH7 Prisms** ( $V_f = 0$ )



Concrete Batch	<b>P</b> _{cr} ¹ (kN)	<b>P</b> _u (kN)	$P_u/P_{cr}$	$\delta_{cr}^{l}$ (mm)	$\delta_u^2$ (mm)	$\delta_u/\delta_{cr}$
B1	63.2	63.2	1.00	0.090	0.090	1.00
B3	66.1	66.1	1.00	-	-	-
В5	58.4	58.4	1.00	0.084	0.084	1.00
Mean	62.6	62.6	-	0.087	0.087	-
Std. Deviation	3.89	3.89	-	0.004	0.004	-
C.V. (%)	6.21	6.21	-	4.60	4.60	-

B2.8 TH8 Prisms (V_f = 1.00 %)



Concrete Batch	<b>P</b> _{cr} ¹ (kN)	<b>P</b> _u (kN)	$P_u/P_{cr}$	$\delta_{cr}^{l}$ (mm)	$\delta_u^2$ (mm)	$\delta_u/\delta_{cr}$
B2	48.2	52.8	1.10	0.098	0.669	6.83
B4	38.3	44.0	1.15	0.091	0.490	5.38
B5	45.6	45.6	1.00	0.084	0.084	1.00
Mean	44.0	47.5	-	0.091	0.414	-
Std. Deviation	5.13	4.69	-	0.007	0.300	-
C.V. (%)	11.66	9.87	-	7.69	72.46	-

 1  development of first crack  2  displacement corresponding to peak load,  $P_{u}$ 

#### **B3:** Uniaxial Tension Tests

#### B3.1 TH3 'Dog-Bones' (V_f = 0.50 %)



(a) Pre-Peak Stress versus Strain Behaviour



(b) Post-Peak Stress versus Crack Width Behaviour

Concrete Batch	P _{cr} (kN)	$f_t$ (MPa)	${\cal E}'_t$ (x 10 ⁻³ mm/mm)	<i>E_{ct}</i> (MPa)
B2	31.2	4.460	0.140	35,600
B4	32.8	4.681	0.175	30,000
Mean	32.0	4.571	0.158	32,800
Std. Deviation	1.13	0.156	0.025	3,960
C.V. (%)	3.53	3.41	15.82	12.07

## B3.2 TH4 'Dog-Bones' (V_f = 1.00 %)



(b) Post-Peak Stress versus Crack Width Behaviour

Concrete Batch	P _{cr} (kN)	$f_t$ ' (MPa)	${\cal E'}_t$ (x 10 ⁻³ mm/mm)	<i>E_{ct}</i> (MPa)
B1	29.4	4.193	0.185	26,000
B3	27.3	3.899	0.147	34,200
В5	29.2	4.167	0.168	28,000
Mean	28.6	4.086	0.167	29,400
Std. Deviation	1.16	0.163	0.019	4,280
C.V. (%)	4.06	3.99	11.38	14.56

## B3.3 TH5 'Dog-Bones' (V_f = 1.50 %)



(b) Post-Peak Stress versus Crack Width Behaviour

Concrete Batch	P _{cr} (kN)	<b>f</b> t' (MPa)	${\cal E}'_t$ (x 10 ⁻³ mm/mm)	<i>E_{ct}</i> (MPa)
B1	32.1	4.582	0.169	29,000
B3	24.6	3.507	0.204	18,500
Mean	28.4	4.045	0.181	24,500
Std. Deviation	5.30	0.760	0.024	7,570
C.V. (%)	18.66	18.79	13.26	30.90

## B3.4 TH8 'Dog-Bones' (V_f = 1.00 %)



(b) Post-Peak Stress versus Crack Width Behaviour

Concrete Batch	P _{cr} (kN)	$f_t$ ' (MPa)	${\cal E'}_t$ (x 10 ⁻³ mm/mm)	<i>E_{ct}</i> (MPa)
B1	27.5	3.930	0.142	35,100
B3	26.2	3.747	0.160	26,300
B5	26.2	3.747	0.150	27,000
Mean	26.6	3.808	0.151	29,500
Std. Deviation	0.75	0.106	0.009	4,890
C.V. (%)	2.82	2.78	5.96	16.58

# **APPENDIX C**: SLAB TEST RESULTS

The following presents a brief outline of Appendix C. Detailed presentations of the test results from the performance slab impact experimental program are presented.

#### C1: Crack Maps

Section C1 of this appendix presents residual crack maps illustrations of the slabs. The crack maps show the progression of damage over the course of the sequential impacts, and have been provided for the top and bottom surfaces of the slabs. For clarity, the impact mass pertaining to each test has been provided in the figure captions.

#### **C2: Midpoint Displacement-Time Histories**

Section C2 of this appendix presents the midpoint displacement-time histories measured from each impact test. Note that the presented displacements represent even displacements and do not include the accumulation residual displacements from prior impacts. The impact mass pertaining to each test has been provided in the figures. The fibre volume fractions and the longitudinal reinforcement ratios comprising each slab are provided in the subsection headings.

#### **C3: Support Reaction Force-Time Histories**

The support reaction force-time histories measured from each impact event have been presented in section C1. Force contributions from the load cells and the tie-down assemblies have been presented separately. Additionally, the total reaction force-time histories resulting from the combined force contributions are presented. The impact mass pertaining to each test has been provided in the figure captions. The fibre volume fractions and the longitudinal reinforcement ratios comprising each slab are provided in the subsection headings.

#### C1: Slab Crack Maps

#### C1.1 Slab TH1 Crack Maps









## C1.2 Slab TH2 Crack Maps



(a) top surface

(b) bottom surface





Figure C1.5-Impact TH2-2 ( $\rho_l = 0.420 \%$ ;  $V_f = 0$ ; m = 180 kg)





## C1.3 Slab TH3 Crack Maps



(a) top surface

(b) bottom surface

Figure C1.8-Impact TH3-2 ( $\rho_l = 0.420$  %;  $V_f = 0.50$  %; m = 180 kg)



Figure C1.9-Impact TH3-3 ( $\rho_l = 0.420$  %;  $V_f = 0.50$  %; m = 210 kg)



Figure C1.10-Impact TH3-5 ( $\rho_l = 0.420$  %;  $V_f = 0.50$  %; m = 240 kg)

## C1.4 Slab TH4 Crack Maps





Figure C1.12-Impact TH4-2 ( $\rho_l = 0.420$  %;  $V_f = 1.00$  %; m = 180 kg)



Figure C1.13-Impact TH4-3 ( $\rho_l = 0.420$  %;  $V_f = 1.00$  %; m = 210 kg)



Figure C1.14-Impact TH4-5 ( $\rho_l = 0.420$  %;  $V_f = 1.00$  %; m = 240 kg)





## C1.5 Slab TH5 Crack Maps



(a) top surface

(b) bottom surface

Figure C1.15-Impact TH5-1 ( $\rho_l = 0.420$  %;  $V_f = 1.50$  %; m = 150 kg)



Figure C1.16-Impact TH5-3 ( $\rho_l = 0.420$  %;  $V_f = 1.50$  %; m = 210 kg)







(a) top surface (b) bottom surface Figure C1.18-Impact TH5-7 ( $\rho_l = 0.420$  %;  $V_f = 1.50$  %; m = 270 kg)





## C1.6 Slab TH6 Crack Maps



Figure C1.20-Impact TH6-1 ( $\rho_l = 0.273$  %;  $V_f = 0$ ; m = 150 kg)



Figure C1.21-Impact TH6-2 ( $\rho_l = 0.273$  %;  $V_f = 0$ ; m = 180 kg)

## D1.7 Slab TH7 Crack Maps



Figure C1.22-Impact TH7-1 ( $\rho_l = 0.592$  %;  $V_f = 0$ ; m = 150 kg)



Figure C1.23-Impact TH7-2 ( $\rho_l = 0.592$  %;  $V_f = 0$ ; m = 180 kg)



Figure C1.24-Impact TH7-3 ( $\rho_l = 0.592$  %;  $V_f = 0$ ; m = 210 kg)

#### C1.8 Slab TH8 Crack Maps





Figure C1.26-Impact TH8-3 ( $\rho_l = 0.592$  %;  $V_f = 1.00$  %; m = 210 kg)


Figure C1.27-Impact TH8-5 ( $\rho_l = 0.592$  %;  $V_f = 1.00$  %; m = 240 kg)



Figure C1.28-Impact TH8-7 ( $\rho_l = 0.592$  %;  $V_f = 1.00$  %; m = 270 kg)





#### **C2: Midpoint Displacement-Time Histories**



## C2.1 Slab TH1 ( $\rho_l = 0.420$ %; $V_f = 0$ ):



C2.2 Slab TH2 ( $\rho_l = 0.420 \%$ ;  $V_f = 0$ ):



Figure C2.2 - TH2 Displacement-Time Histories

C2.3 Slab TH3 ( $\rho_l = 0.420$  %;  $V_f = 0.50$  %):



Figure C2.3 – TH3 Displacement-Time Histories

C2.4 Slab TH4 ( $\rho_l = 0.420$  %;  $V_f = 1.00$  %):



Figure C2.4 – TH4 Displacement-Time Histories



Figure C2.4 (*continued*) – TH4 Displacement-Time Histories ( $\rho_l = 0.420$  %;  $V_f = 1.00$  %)

C2.5 Slab TH5 ( $\rho_l = 0.420$  %;  $V_f = 1.50$  %):



Figure C2.5 – TH5 Displacement-Time Histories



Figure C2.5 (*continued*) – TH5 Displacement-Time Histories ( $\rho_l = 0.420$  %;  $V_f = 1.50$  %)

C2.6 Slab TH6 ( $\rho_l = 0.273$  %;  $V_f = 0$ ):



Figure C2.6- TH6 Displacement-Time Histories

C2.7 Slab TH7 ( $\rho_l = 0.592$  %;  $V_f = 0$ ):



Figure C2.7 – TH7 Displacement-Time Histories

C2.8 Slab TH8 ( $\rho_l = 0.592$  %;  $V_f = 1.00$  %):



Figure C2.8 - TH8 Displacement Time-Histories



Figure C2.8 (*continued*) – TH8 Displacement-Time Histories ( $\rho_l = 0.592$  %;  $V_f = 1.00$  %)

#### **C3: Support Reaction Force-Time Histories**



# C3.1 Slab TH1 ( $\rho_l = 0.420$ %; $V_f = 0$ ):



Figure C3.1 – TH1 Support Reaction-Time Histories







Figure C3.2 - TH2 Support Reaction-Time Histories

C3.3 Slab TH3 ( $\rho_l = 0.420$  %;  $V_f = 0.50$  %):









Figure C3.3 (continued) – TH3 Support Reaction-Time Histories ( $\rho_l = 0.420$  %;  $V_f = 0.50$  %)

### C3.4 Slab TH4 ( $\rho_l = 0.420$ %; $V_f = 1.00$ %):





Figure C3.4 – TH4 Support Reaction-Time Histories



Figure C3.4 (*continued*) – TH4 Support Reaction-Time Histories ( $\rho_l = 0.420$  %;  $V_f = 1.00$  %)







C3.5 Slab TH5 ( $\rho_l = 0.420$  %;  $V_f = 1.50$  %):



Figure C3.5 – TH5 Support Reaction-Time Histories















(j) Impact Event TH5-10 (m = 300 kg)

Figure C3.5 (*continued*) – TH5 Support Reaction-Time Histories ( $\rho_l = 0.420$  %;  $V_f = 1.50$  %)







Figure C3.6 – TH6 Support Reaction-Time Histories

C3.7 Slab TH7 ( $\rho_l = 0.592$  %;  $V_f = 0$ ):



(c) Impact Event TH7-3 (m = 210 kg)

Figure C3.7 – TH7 Support Reaction-Time Histories

C3.8 Slab TH8 ( $\rho_l = 0.592$  %;  $V_f = 1.00$  %):





Figure C3.8 – TH8 Support Reaction-Time Histories









Figure C3.8 (*continued*) – TH8 Support Reaction-Time Histories ( $\rho_l = 0.592$  %;  $V_f = 1.00$  %)